as n approaches infinity, the freedom of this motion becomes perfect. This is the free motion — the protected mode — we sought.

This freedom is subtly connected to the existence of two equivalent ground states. To see this, we repeat the above procedure using the opposite ground state with left-tilting rods (Fig. 1d). Again we rotate the rightmost to the right by one degree. As before, the rightmost length will increase unless the next-to-last node also moves. However, now the angles have reversed, and the next-to-last node must rotate more than one degree. This amplification continues as we move across the lattice, so that the leftmost node must rotate exponentially more than one degree. The fixed link on the left must now stretch substantially. There is no free motion.

This cute example embodies a far-reaching topological principle, though at the moment no topology is apparent. To explain the connection, we must choose a way to quantify small deformations of the system, capable of capturing the difference between left-leaning and right-leaning states. An appropriate choice is the matrix of proportionality constants relating small node movements to the resulting length changes of the links. Any topological property that entails free (protected) motion is necessarily a property of this response matrix.

How can the existence of two equivalent ground states place constraints on this matrix? And how might these constraints be dictated by topology, which requires smooth deformations? Clearly, to make use of topology, we must find a way to deform the system smoothly from one ground state to the other. We could do this by simply pushing all the rods in Fig. 1a to the left until the tilt flips from right to left. However, such a push would take us far from the domain of the linear response described by our matrix. What we need is a continuous process that carries us from the matrix of the right-tilting state to that of the left-tilting state.

The transformation required amounts to translating the links one site to the right. This translation makes the original matrix resemble that of the left-tilting ground state. Using it, one can create a smooth transformation from the right-tilting to the left-tilting matrix, and back to the right-tilting matrix, without ever reversing direction — like a loop threading a bagel. Although we cannot continuously change from the left-tilting to the right-tilting state, what we can do is more important: we can pass smoothly from the response function of one state to that of the other.

Drawing on prior work in lattice field theory[3], Kane and Lubensky showed how to generalize the elements of the zigzag model to higher-dimensional lattices[1]. As with the zigzag lattice, they must be characterized by a 'polarization vector' that is analogous to the rightward shift in the zigzag. This picture reveals another mode paired with each protected free mode, dubbed a state of self-stress. In Fig. 1c, the state of self-stress is on the left side: a compressive stress in the fixed link dies away exponentially on successive links.

Paulose *et al.*[2] took this development a significant step further. They showed how the previous formalism[1,3] may be generalized from perfect lattices to lattices with defects. Lattice defects such as the familiar dislocations of crystallography[4] have a well-known topology of their own. One may create a dislocation in any periodic lattice, and ask how it might affect the free and self-stressed modes. The authors answered this question elegantly by generalizing the Kane–Lubensky methods to a finite lattice with a dislocation[2]. The number of topological boundary modes is simply the cross product of the Burgers vector — or lattice displacement — of the defect, and the polarization vector of the topological lattice without defects. When this product is negative, there is a self-stressed mode at the boundary paired with a free mode localized near the defect. When there are two compensating dislocations, the boundary modes disappear and the free and self-stressed modes are each localized at one of the defects.

These mechanical free modes have a special significance for soft condensed matter. Recently, attention has turned to constructing soft materials defined as discrete networks. Examples are jammed granular packs[5], periodic cavity arrays that buckle asymmetrically under stress[6,7] and origami structures[8]. The aim is to channel applied local forces into specific global pathways that create an organized deformation or stress across the material — precisely what the reported topological free modes[1,2] achieve. ❐

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ROSETTA MISSION

When the dust has settled

The Rosetta orbiter following Comet 67P has captured not only the public imagination but also actual dust grains from the comet's nucleus, revealing their composition, morphology and strength.

David Jewitt

Comets attract enormous scientific interest as carriers of primitive materials, including ices and organics, from the dawn of the Solar System. Formed 4.5 billion years ago outside the snow line (the distance from the Sun beyond which water was cold enough to exist as a solid rather than as a gas), they have been stored ever since in one of two enormous but dimly perceived reservoirs called the Kuiper belt and the Oort cloud. The former is a disk-shaped region beyond Neptune that holds a few billion comets and supplies the so-called short-period comet population. The latter is a spherical cloud that extends a quarter of the way to the next nearest stars and holds 100 billion comets, some of which dip into the planetary region of the Solar System during their long-period orbits. Study of the comets and their deep-freeze storage reservoirs has become a central focus of modern planetary science and provides the motivation behind the European Space Agency's on-going and spectacular Rosetta mission to Comet 67P. Writing in *Nature*, Rita Schulz *et al.*[1] report that the COSIMA instrument on Rosetta has detected dust eroded from the rubble mantle of Comet 67P.

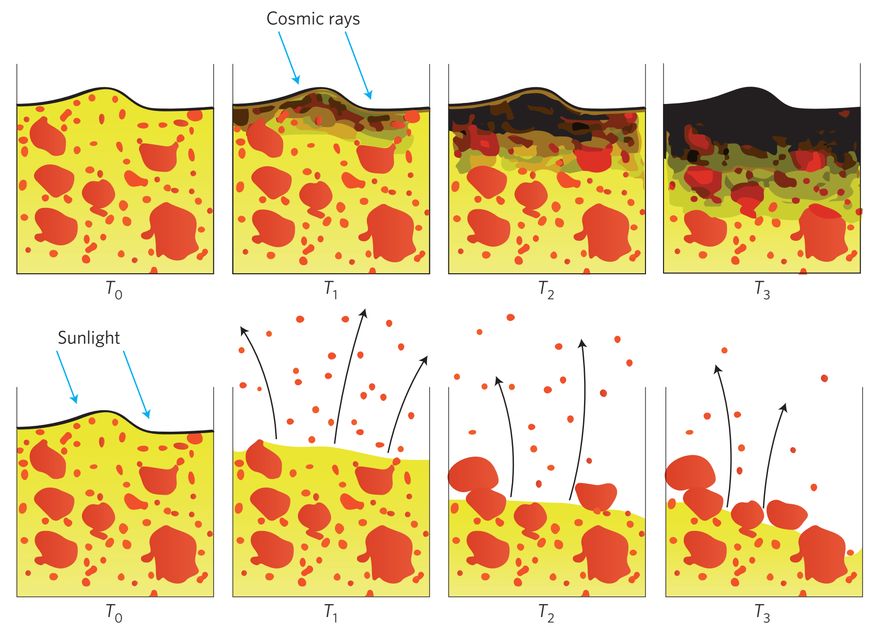

Figure 1 | From dust to dust. **a**, Cross-section of the irradiation mantle of a cometary nucleus. Ice (yellow) and dust (red) are exposed to the flux of cosmic rays. Over time (T_0 to T_3 is about a billion years) a surface layer with a thickness of roughly one metre is carbonized, forming an irradiation mantle (black). **b**, Cross-section of the rubble mantle of a cometary nucleus. Ice sublimates due to the heat of the Sun, dragging out small particles of dust and leaving behind large dust grains that are too heavy to lift. They accumulate over time (T_0 to T_3 is a few years) to form a surface mantle.

In the canonical 'dirty snowball' model — established by legendary planetary astronomer Fred Whipple in 1950 — the cometary nucleus is a kilometre-sized, 50:50 conglomerate of ices and refractory material ('dust' made largely of silicates and carbonaceous material) accreted in the early days of the Solar System[2]. When a comet approaches the Sun, the ices sublimate to produce a wind that drags out dust particles, giving the comets their distinctive fuzzy appearances. Inside the orbit of Jupiter, sublimation can be very rapid and the lifetimes of active comets are correspondingly short; a few tens of thousands of years for comets like 67P.

Perhaps with visions of Alpine snowbanks in mind, early researchers assumed that the nuclei of comets would be highly reflective[3]. Starting in the 1980s, however, observations began to show that the nuclei are instead as dark as crumbled charcoal, reflecting only a few per cent of the incident solar radiation. This was soon explained in terms of a surface dust crust[4,5], somewhat confusingly called a mantle by comet aficionados.

There are two kinds of cometary dust mantle. Comets still in the Kuiper belt and Oort cloud are bombarded by cosmic rays, sputtering the material and severing the bonds in cometary ices, freeing hydrogen to escape. What's left behind is a chemically complex, high molecular-weight residue rich in carbon. The penetration depth of GeV atomic nuclei in ice is about a metre. Over the course of a billion years or so a carbonized metre-thick irradiation mantle should develop (Fig. 1a). Indeed, many of the smaller Kuiper belt objects have dark surfaces, like the comet nuclei, and show red colours sometimes compared to irradiated organic materials[6,7].

Comet 67P is a short-period escapee from the Kuiper belt but it has already lost so much mass during previous orbits around the Sun that its irradiation mantle has long since been ejected or buried by the build-up of surface debris. Instead, it has a 'rubble mantle' formed from exhumed dust particles that are too large (or too sticky) to escape, through gas drag, from the surface of the nucleus[4,6,8] (Fig. 1b). Heat from the Sun penetrates only a few diurnal thermal skin depths into the surface, a distance of perhaps 10 cm or 20 cm on 67P. A rubble layer this thick or thicker can grow in a few months or years[6,8] and would substantially impede the sublimation of icy material deeper down. Narrow jets that are commonly observed on comets appear to indicate constriction of the gas flow by inert rubble mantles.

The COSIMA instrument on-board Rosetta uses a plate to collect dust particles expelled from the nucleus and analyses them with a microscope and a time-of-flight mass spectrometer. Schulz *et al.*[1] report that particles scoured from the mantle — each particle is a few tenths of a millimetre across and strikes the instrument at only 1 to 10 m s^{-1} — have a 'bunch-of-grapes' morphology so fragile that they break-up on impact. Finding evidence for a rubble mantle on 67P is not by itself new or exciting, but the COSIMA investigators make an additional claim: they report a high abundance of sodium in the collected particles, similar to values inferred in some meteors of likely cometary origin. Sodium is a volatile element easily lost from a mineral by heating. The COSIMA dust is much less altered by the collection process than the dust returned by NASA's Stardust mission[9], which had a relative velocity of 6 km s^{-1} — high enough to crush, melt and vaporize the particles, confounding measurements of morphology and volatile abundance.

The combination of high sodium abundance, a bunch-of-grapes morphology and an association with a comet rings three loud bells. Since the 1970s, Donald Brownlee and others have collected strange extraterrestrial dust particles from the stratosphere using high-altitude aeroplanes[10]. It is widely suspected that the Brownlee particles are pieces of comet that have drifted into the atmosphere of the Earth, although proof has been lacking. The COSIMA measurement moves this suspicion one step closer to a certainty. ❐

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