



The Influences of Hydrogen–Silicate–Iron Miscibility on the Demographics of Sub-Neptunes and Super-Earths

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Received 2026 March 4; revised 2026 June 17; accepted 2026 June 26; published 2026 July 24

Abstract

Models based on variable miscibility among hydrogen, molten silicate, and molten iron, coupled with atmospheric escape, can reproduce the observed occurrence density structure of sub-Neptunes and super-Earths in mass–radius space. The models are also consistent with the radius gap and the observed radius–period relationship exhibited by these planets. The degree of overlap between the predicted and observed planetary occurrences suggests that hydrogen–silicate–iron miscibility may serve as a unifying concept for the formation and evolution of these planet classes. The well-defined equilibrium conditions at the boundary between supercritical magma oceans and the overlying hydrogen-rich envelopes are important features of the models. Planets formed with less than $\sim 1\%$ hydrogen by mass develop discrete, terrestrial-like metallic cores, while those accreting greater hydrogen concentrations are predicted to have fully miscible interiors and no discrete metal cores. Hydrogen–silicate–iron miscibility provides an overarching explanation for the full range of sub-Neptune and super-Earth architectures based on the accreted hydrogen mass fraction and the phase equilibria governing silicate, iron metal, and H_2 miscibility.

Unified Astronomy Thesaurus concepts: [Exoplanet formation \(492\)](#)

1. Introduction

Sub-Neptunes are apparently the most abundant planet type in the Galaxy (e.g., F. Fressin et al. 2013; B. J. Fulton et al. 2017). These planets are characterized by bulk densities intermediate between those of rocky planets and Neptune-like planets. Their low densities relative to compressed rock and metal suggest that sub-Neptunes may be mixtures of roughly equal-mass fractions of rock and water (e.g., J. J. Fortney et al. 2007; L. Zeng et al. 2019) or molten/partially molten interiors beneath hydrogen-rich primary atmospheres (Y. Chachan & D. J. Stevenson 2018). The H_2 -rich envelopes in this interpretation constitute percent levels of the planet masses in order to match their bulk densities (e.g., J. L. Bean et al. 2021).

There is a rich history of attempting to infer the chemistry and structure of sub-Neptunes from their bulk densities. Applications of equations of state (EoSs) for silicate, iron metal, water, and H_2 have been used to invert mass and radius data for the compositions and mass fractions of various layers comprising individual planets. These previous studies account for the compression of rock and metal (e.g., D. Valencia et al. 2006; S. Seager et al. 2007) and the inclusion of H_2 and water layers (e.g., C. Sotin et al. 2007; L. A. Rogers & S. Seager 2010; E. D. Lopez & J. J. Fortney 2014; L. Zeng et al. 2016; A. Aguichine et al. 2021; M. C. Nixon & N. Madhusudhan 2021; A. Vazan et al. 2022). Probabilistic approaches have been applied to make clear where degeneracies can and cannot be broken (e.g., A. Wolfgang & E. Lopez 2015; C. Dorn et al. 2017).

Dense H_2 -rich atmospheres can limit the cooling of sub-Neptunes such that initially molten interiors stay molten for

Gyr timescales (M. Ikoma & H. Genda 2006; E. D. Lopez & J. J. Fortney 2014; S. Ginzburg et al. 2016). As a result, there has been a recent focus on the interaction between hydrogen atmospheres and underlying magma oceans (E. S. Kite et al. 2019; J. L. Bean et al. 2021; H. E. Schlichting & E. D. Young 2022; A. Vazan et al. 2022; S. Charnoz et al. 2023; W. Misener et al. 2023; E. D. Young et al. 2024, 2025; J. G. Rogers et al. 2025b; A. Werlen et al. 2025a, 2025b). Experimental verification for the implications of H_2 –silicate–metal interactions deduced from the extrapolation of chemical thermodynamics and ab initio molecular dynamics is just now becoming available (H. W. Horn et al. 2025; F. Miozzi et al. 2025).

Super-Earths are evidently composed primarily of rock and metal. They are most often thought to represent the stripped cores left behind by the nearly complete escape of the primary atmospheres comprising sub-Neptunes (J. E. Owen & Y. Wu 2013, 2017; E. D. Lopez & J. J. Fortney 2014; V. Van Eylen et al. 2018; A. Gupta & H. E. Schlichting 2019). The key features of these arguments are the existence of a radius valley, or gap, between super-Earths and sub-Neptunes that is near $1.8M_{\oplus}$ farther from the host star (longer orbital periods) and $2.0M_{\oplus}$ nearer to the star (shorter orbital periods) (B. J. Fulton et al. 2017; V. Van Eylen et al. 2018; A. Gupta & H. E. Schlichting 2019). An alternative model suggests that with an appropriately broad initial mass function for rocky/metal cores and accretion late in the evolution of a protoplanetary disk, the period-dependent radius gap can be a result of planet accretion (E. J. Lee & N. J. Connors 2021).

A result of a deeper understanding of the physical chemistry of hydrogen, silicate, and metal interactions is that the traditional view of sub-Neptunes and their potential super-Earth descendants as having purely metal cores and silicate mantles, with or without surviving primary hydrogen-rich envelopes, may not be universally correct. Rather, at relatively moderate temperatures and pressures in the context of these planets, H_2 and silicate become entirely miscible



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(T. Gilmore & L. Stixrude 2026), with complete miscibility defining the boundary between a supercritical magma ocean and overlying H_2 -rich envelope (E. D. Young et al. 2024). In addition, within the interior (sometimes referred to as the “core,” to distinguish it from the overlying envelope), H_2 , silicate, and iron metal are predicted to become fully miscible (E. D. Young et al. 2025).

In this paper, we seek to answer the question as to whether or not the newly recognized physical chemistry of hydrogen–silicate–metal planets is consistent with the populations of sub-Neptunes and super-Earths. The paper is organized as follows: our epistemological approach for constructing model planets for comparisons with observed planets is presented in Section 2; the planet models are described in Section 3; the random sampling of planet models is explained in Section 4; the resulting model planet population is compared with observed planets in Section 5; the implications of the results are discussed in Section 6; and our conclusions are summarized in Section 7.

2. Epistemological Approach

Two different approaches are often used when attempting to explain the observable properties of sub-Neptunes and super-Earths: (1) *forward evolutionary models*, in which a planet’s thermal state and atmospheric mass fraction are evolved from specified initial conditions of the specific entropy, mass, and mass fraction of volatile components (e.g., E. D. Lopez & J. J. Fortney 2014; J. E. Owen & Y. Wu 2017); and (2) *inverse statistical inference*, in which emphasis is placed on constraining the present-day planetary properties necessary to explain the observables (e.g., L. A. Rogers & S. Seager 2010; A. Wolfgang & E. Lopez 2015; C. Dorn et al. 2017). These approaches differ not only in methodology and philosophy, but also in computational cost and sensitivity to the inferred initial conditions.

A limitation of forward models is their computational cost. High-fidelity evolution models include detailed solutions for energy transfer—e.g., $dE_p/dt = -L_{\text{int}}$, where E_p is the total thermal and gravitational potential energy of the planet and L_{int} is the intrinsic net luminosity leaving the body—as well as hydrodynamic or energy-limited atmospheric escape. The forward integrations can require time stepping through tens of thousands of model calls per planet, in some cases accelerated by using precomputed grids or libraries that can be used for interpolation.

Forward models place an emphasis on the time evolution from an initial state. However, these initial states are plagued by uncertainties surrounding the retention of heat during the accretion process (E. D. Young et al. 2024). In addition, sub-Neptunes undergo rapid thermal evolution, losing most of their primordial heat within their first $10\text{--}10^2$ Myr (E. D. Lopez & J. J. Fortney 2014). Although these planets cool rapidly at early times, evidence of their initial conditions can persist until late in their evolution (e.g., R. Tejeda Arevalo et al. 2025). However, these radii differences appear to dwindle to a few percent at very late ages and disappear entirely by ~ 10 Gyr. Conversely, while forward models attempt to determine what planet results from a set of inferred initial conditions, inverse methods instead place the emphasis on the extant underlying chemistry and structure consistent with observable features like mass and radius.

Primarily because of the computational costs associated with forward evolution models in the present case, but also because of uncertainties surrounding the completeness of the initial conditions, we adopt an inverse approach in this paper. Because evolutionary pathways are not uniquely recoverable from present-day observables, the inverse problem is fundamentally probabilistic. The goal is to infer the minimum number of plausible initial conditions and evolutionary histories that are compatible with the present-day observables. This approach minimizes assumptions about initial conditions—e.g., the total specific entropy that sets the central core temperature but is effectively erased on 100 Myr timescales—and computational costs, which are limiting in this application.

Our approach is to draw random values for variables that prescribe the states of sub-Neptune planets that include, where appropriate, complete miscibility in their interiors, using a range for each variable that is consistent with the likely ages matching the observed planet ages. We use those variables to construct model populations of sub-Neptunes. We then compute the probability that each model planet could have retained its primordial H_2 -rich atmosphere given the host-star environment and age assigned to it. If the probability is low, then this planet is stripped of its primary atmosphere, leaving behind a core that is now a super-Earth. We compare the resulting demographics of model planets to the observed population of super-Earths and sub-Neptunes.

3. Planet Models

3.1. Planet Structure

Many of the methods for calculating the planet models used here have been described previously (J. G. Rogers et al. 2025b; E. D. Young et al. 2025). However, there are some modifications to the prior published work, and so the models are explained here. To derive the core,¹ we solve the system of equations (e.g., S. Seager et al. 2007)

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad (1)$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}, \quad (2)$$

and

$$\left(\frac{dT}{dr}\right)_s = -\frac{\gamma(\eta) T}{K_S(\eta)} \rho g, \quad (3)$$

where m is the mass contained within the radius r , ρ is the mass density, $\gamma(\eta)$ is the Grüneisen parameter that varies with P and T through the compressibility factor $\eta = \rho/\rho_0$, and $K_S(\eta)$ is the isentropic bulk modulus. Here, one assumes the core is fully convective, with a limiting isentropic dT/dP gradient. In practice, we evaluate the bulk modulus using $K_S(\eta) \equiv \eta d(P(\eta, T_S))/d\eta$, where $P(\eta, T_S)$ is the total isentropic pressure along the adiabat with temperature T_S . Numerically integrating Equations (1) through (3) through one or more layers (with or without a metal core) yields a density and temperature profile for the planet interior.

¹ Throughout this paper, we will follow the convention in the astronomical fields and use the term “core” to mean everything interior to the envelope or atmosphere of the planet. In some cases, modifiers like “metal” will be used to signify a separate phase in the interior, consistent with geophysical usage.

Our use of the adiabat is predicated on a molten interior throughout. We estimate the depression of the MgSiO_3 solidus by dissolved H_2 using the ideal cryoscopic relation:

$$\Delta T_m = -\frac{RT_m^2}{\Delta H_{\text{fus}}} \ln x_{\text{MgSiO}_3}, \quad (4)$$

where x_{MgSiO_3} is the mole fraction of MgSiO_3 in the melt. For representative binodal conditions near 4 GPa, an H_2 content of $\sim 2\text{wt}\%$ corresponds to $x_{\text{MgSiO}_3} \simeq 0.50$. Taking $T_m \simeq 3200$ K and $\Delta H_{\text{fus}} \simeq 75$ kJ mol $^{-1}$ (L. Stixrude & B. Karki 2005; L. Stixrude 2014) gives $\Delta T_m \simeq 800$ K. Because $\Delta T_m \propto T_m^2/\Delta H_{\text{fus}}$ and T_m increases strongly with pressure, the melting-point depression is expected to remain large, and likely increase, deeper in the planets. The depression of the melting temperature with mixing ensures that the temperatures along the adiabats in our models are above the melting temperatures (Y. Fei et al. 2021). Similar arguments apply to Fe mixed with hydrogen, for which dissolved H also stabilizes the liquid relative to the solid and lowers the liquidus.

For the structure of the envelope, we integrate numerically the hydrostatic and mass conservation equations as in the interior (Equations (1) and (2)), using the tabulated function from G. Chabrier et al. (2019) for the EoS. The temperature structure of the envelope is obtained by numerically integrating the coupled hydrostatic, mass conservation, and energy transport equations outward from the magma ocean surface, followed by an iterative inward integration from the optically thin outer atmosphere. Throughout most of the envelope, the energy transport is convective, and the temperature gradient is therefore taken to be the convective (pseudo-adiabatic) gradient:

$$\frac{dT}{dr} = -\nabla T_{\text{conv}}, \quad (5)$$

where ∇T_{conv} is evaluated using a moist pseudo-adiabat appropriate for an H_2 –silicate vapor mixture (R. J. Graham et al. 2021), scaled to the H_2 EoS (G. Chabrier et al. 2019). Above the convective layer where the gas is still collisional, radiative diffusion applies:

$$\nabla T_{\text{rad}} = -\frac{3\kappa\rho L_{\text{int}}}{64\pi\sigma T^3 r^2}, \quad (6)$$

where κ is the local opacity, ρ is the density, and L_{int} is the intrinsic luminosity carried upward from the interior. The envelope transitions smoothly into a radiative regime by relaxing the temperature gradient with Eddington’s two-stream solution for a plane-parallel gray atmosphere (T. Guillot 2010):

$$T^4(\tau) = \frac{3}{4} T_{\text{int}}^4 \left(\tau + \frac{2}{3} \right) + \frac{3}{4} T_{\text{eq}}^4 \left[\frac{2}{3} + \frac{2}{3\gamma_{\text{vis}}} + \left(\frac{\gamma_{\text{vis}}}{3} - \frac{2}{3\gamma_{\text{vis}}} \right) e^{-\gamma_{\text{vis}}\tau} \right], \quad (7)$$

where τ is the optical depth measured downward from the top of the atmosphere, T_{eq} is the equilibrium temperature set by stellar irradiation, T_{int} is the intrinsic temperature associated with the interior luminosity, and γ_{vis} in this context is the visible-to-thermal opacity ratio. We impose the conservation of intrinsic luminosity, rather than a constant intrinsic flux as

assumed in the plane-parallel equations, by accounting for the area dependence of the intrinsic flux $f_{\text{int}}(r) = \sigma T_{\text{int}}^4(r)$ such that $T_{\text{int}}(r) = T_{\text{int},s}(r_s/r)^{1/2}$, where the subscript “s” signifies evaluation just above the supercritical magma ocean. At large infrared optical depth ($\tau \gg 1$), Equation (7) approaches the radiative-diffusion limit, yielding a radiative temperature gradient consistent with Equation (6) and thus ensuring consistency with deep-envelope energy transport. This approach affords a smooth and numerically stable transition from the deep convective adiabat to the irradiated upper atmosphere.

3.2. Luminosity

The intrinsic luminosities of sub-Neptunes are largely unknown and model-dependent. Most often, evolution models specify an initial entropy and intrinsic luminosity, L_{int} , based on inferences about the retention of heat during accretion. However, this approach does not guarantee self-consistency; a specified L_{int} may exceed, or fall short of, the flux the atmosphere can transmit given its opacity structure and boundary conditions, leading to either interior heating or unphysical cooling timescales. A closure for this problem, and thus self-consistent values for L_{int} , can be obtained from a consideration of boundary layer effects.

While the bulk of the lower region of the hydrogen-rich envelope is convective, a third layer at the base of the atmosphere arises from a boundary layer between the supercritical melt and the overlying convective region in the envelope (G. Ahlers et al. 2009), augmented in many cases by the hindrance of convection due to the mass loading of heavy elements at relatively high temperatures (W. Misener et al. 2023). The lower boundary layers restrict the intrinsic upward luminosities that comprise heat transfer from the core to the atmosphere, forming a bottleneck.

The minimum thickness of the basal boundary layer separating the interior (core) and the overlying envelope, δ , comes from consideration of the boundary for Rayleigh–Bénard convection in the envelope. The width of the boundary layer is controlled by the vigor and scale of the convection and is given by scalings similar to

$$\delta \sim nh (Ra/Ra_c)^{-1/3}, \quad (8)$$

where h is a characteristic convecting length scale, taken to be some multiple n of the pressure scale height evaluated at the base of the atmosphere; Ra is the Rayleigh number; and Ra_c is the critical Rayleigh number for the convecting H_2 -rich envelope (e.g., S. Grossmann & D. Lohse 2000; R. S. Long et al. 2020). The Rayleigh number is computed as

$$Ra = \frac{g \alpha \Delta T h^3}{\nu \kappa}, \quad (9)$$

where h is here taken to be $5H$ for a scale height H , ΔT is the temperature contrast across the layer, ν is the kinematic viscosity, κ is the thermal diffusivity, and α is the thermal expansivity. The use of the adiabat for ΔT , rather than superadiabatic temperatures, likely overestimates Ra , resulting in underestimates for the widths of the boundary layers (see below). The kinematic viscosities for H_2 are estimated here

using a Sutherland law fit for the dynamic viscosities η :

$$\eta(T) = \eta_0 \left(\frac{T}{T_0} \right)^{3/2} \frac{T_0 + S_T}{T + S_T}, \quad (10)$$

with the reference viscosity η_0 at reference temperature T_0 and the Sutherland temperature S_T for H_2 from J. Braun et al. (2018). The kinematic viscosities are then

$$\nu = \frac{\eta}{\rho}. \quad (11)$$

The thermal expansivities required to evaluate Ra are obtained using

$$\alpha = -\frac{1}{T} \left(\frac{\partial \log \rho}{\partial \log T} \right)_p, \quad (12)$$

based on the EoS tables for hydrogen from G. Chabrier et al. (2019). We adopt a thermal diffusivity κ of $1.0 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ for the purpose of evaluating approximate values for the Rayleigh number. For nominal parameters relevant for sub-Neptunes in this study—e.g., $\Delta T = 3000 \text{ K}$, surface pressure $P_s = 1 \text{ GPa}$, planet mass $M_p = 6M_\oplus$, and density $\rho = 47 \text{ kg m}^{-3}$ —one obtains $Ra \simeq 10^{32}$.

The critical Rayleigh number depends on the rotation rate, among other factors (J. M. Aurnou et al. 2020), through the Ekman number ($Ek = \nu/(2\Omega h^2)$, where ν is the kinematic viscosity, Ω is the rotation period of the body, and h is the relevant height. A commonly used scaling in the limit where $Ek \rightarrow 0$ is (S. Chandrasekhar 1961)

$$Ra_c = 8.7 Ek^{-4/3}. \quad (13)$$

The Ekman number for a $6M_\oplus$ sub-Neptune with the atmosphere used to evaluate Ra described above, where $\nu = 9.6 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ and $H \sim 10^5 \text{ m}$, for tidally locked rotation periods from 10 days to 100 days, is 2×10^{-15} to 2×10^{-14} , yielding R_c values of 3×10^{20} to 2×10^{19} , respectively. When paired with the corresponding Rayleigh numbers of 2×10^{32} , Ra/Ra_c ranges from 10^{11} to 10^{13} and the boundary layer thicknesses implied by Equation (8) are of order 20 to 100 m. We can combine these thicknesses with opacities to derive the optical depth for the radiative boundary layer: $\tau_{BL} = \kappa \rho \delta$.

The intrinsic luminosity L_{int} is determined from the blackbody emission $L_{\text{bb}} = 4\pi r_s^2 \sigma T_s^4$ using the ability of the magma ocean to radiate through the boundary layer immediately above its surface, which for a gray atmosphere is

$$L_{\text{int}} = L_{\text{bb}} \left(\frac{4}{3(\tau_{BL} + 1)} \right). \quad (14)$$

The intrinsic temperature at the surface satisfies

$$L_{\text{int}} = 4\pi r_s^2 \sigma T_{\text{int},s}^4, \quad (15)$$

This formulation allows for self-consistency between the surface of the magma ocean and the upper atmosphere, accommodating the link with intrinsic luminosity in the regime of vigorous convection. Rather than resolving the temperature gradient in this often very narrow boundary, its influence as a

governor on the radiative flux is captured through Equation (14).

The basal radiative layer can be further stabilized and enhanced in elevation by the molecular weight gradients that attend the interaction between the supercritical magma ocean and the overlying envelope. As a result, the boundary between the surface of the magma ocean and the convective layer in the envelope can be both pronounced and explicitly resolvable in our models. Rather than allowing arbitrarily large intrinsic luminosities that would dynamically destroy stratification, at least temporarily, we adopt a quasi-static closure in which a Ledoux-stable layer, when permitted thermodynamically, forms, persists, and therefore acts as a finite thermal resistance that limits the transmitted flux. Thermal bottlenecks created by compositional gradients in the interiors of sub-Neptunes have been described by A. Vazan et al. (2018) and R. Tejada Arevalo et al. (2025).

Stability against convection in the presence of composition gradients is governed by the Ledoux criterion:

$$\nabla_{\text{rad}} > \nabla_{\text{ad}} + \frac{\phi}{\delta} \nabla_\mu, \quad (16)$$

where

$$\nabla_\mu \equiv \frac{d \ln \mu}{d \ln P}, \quad (17)$$

$$\delta \equiv - \left(\frac{\partial \ln \rho}{\partial \ln T} \right)_{P,\mu}, \quad (18)$$

and

$$\phi \equiv \left(\frac{\partial \ln \rho}{\partial \ln \mu} \right)_{P,T}. \quad (19)$$

Here, μ is the mean molecular weight. A positive molecular weight gradient ($\nabla_\mu > 0$) increases the critical temperature gradient required for convective instability by the factor $(\phi/\delta)\nabla_\mu$. The ratio (ϕ/δ) is unity for an ideal gas and remains of order unity for reasonable values for compressibility factor Z , so in practice we simplify the Ledoux criterion (Equation (16)) to $\nabla_{\text{rad}} > \nabla_{\text{ad}} + \nabla_\mu$. Where the Ledoux-stable region spans a finite radial extent, it further limits L_{int} for the overlying atmosphere layers. The intrinsic luminosity is limited by the requirement that the layer remain marginally Ledoux-stable. In this radiative layer, the upward energy flux is given by the diffusion approximation

$$F_{\text{rad}} = -\frac{16\sigma T^3}{3\kappa\rho} \frac{dT}{dr}, \quad (20)$$

which may be written in terms of the dimensionless gradient ∇ as

$$F_{\text{rad}} = \frac{4\sigma T^4}{3\kappa\rho H} \nabla, \quad (21)$$

where $H = P/(\rho g)$ is the pressure scale height and $\nabla \equiv d \ln T / d \ln P$. The maximum radiative flux consistent with stability against convection is obtained by evaluating this expression at the largest temperature gradient permitted by the

Ledoux criterion:

$$\nabla_{\max} = \nabla_{\text{ad}} + \frac{\phi}{\delta} \nabla_{\mu}. \quad (22)$$

In the radiative layer, we therefore have

$$\nabla_{\text{rad}} = \nabla_{\max} = \frac{3\kappa PL}{64\pi\sigma GMT^4}, \quad (23)$$

leading to a definition for the maximum luminosity that can be transported without violating Ledoux stability:

$$L_{\text{Ledoux}} = \left(\frac{64\pi\sigma GMT^4}{3\kappa P} \right) \nabla_{\max}, \quad (24)$$

evaluated at a representative location near the top of the Ledoux-stable region. The intrinsic luminosity is then capped such that $L_{\text{int}} = \min(L'_{\text{int}}, L_{\text{Ledoux}})$, where L'_{int} is the value in the absence of the stable boundary. This upper limit is applied iteratively during the solution procedure until a self-consistent thermal structure satisfying Ledoux stability is obtained. Any larger luminosity would require a radiative temperature gradient steeper than $\nabla_{\max}$, causing ∇_{rad} to exceed the Ledoux stability limit. In that case, the stabilizing molecular weight gradient would no longer suppress convection, and the layer would become convectively unstable.

Expressing the radiative flux as a finite difference across the Ledoux-stable region, we have

$$L_{\text{Ledoux}} \simeq 4\pi r_{\mu,\max}^2 \frac{4\sigma}{3} \frac{T_{\text{lower}}^4 - T_{\text{upper}}^4}{\Delta\tau_{\text{Ledoux}}}, \quad (25)$$

which makes explicit that the transmissible luminosity is controlled by the integrated optical depth thickness across the inhibited region $\Delta\tau_{\text{Ledoux}}$. Here, T_{lower} and T_{upper} are the temperatures at the lower and upper boundaries of the layer, and $r_{\mu,\max}$ is a representative radius within the region where ∇_{μ} is maximized. Equation (25) therefore represents the integrated radiative throughput of the Ledoux-stable layer and provides a global maximum for the intrinsic luminosity. It also avoids numerical oscillations that can arise from high-frequency layer-by-layer adjustments of the temperature gradient.

Molecular weight gradients specify where stratification is stable against overturn. In the presence of a phase change, an additional threshold must obtain for stability, due to the thermal effects of the enthalpy of reaction. Accordingly, we include a microphysical inhibition threshold, following S. Markham et al. (2022), based on a critical “heavy-species” mole-fraction threshold x_{inhib} :

$$x_{\text{inhib}} = \left[\left(\frac{\mu_{\text{melt}} \Delta H}{RT} - 1 \right) (\epsilon - 1) \right]^{-1}, \quad (26)$$

where μ_{melt} is the mean molecular weight of the condensed phase, $\epsilon \equiv \mu_{\text{vap}}/\mu_{\text{H}_2}$, where μ_{vap} is the mean molecular weight of the heavy condensing vapor species (e.g., Mg and SiO); and ΔH is the effective latent heat per unit mass for condensation. Physically, x_{inhib} encodes stable stratification in the presence of the phase change. In practice, the Ledoux-stable inhibited region is activated only when both a stabilizing molecular weight gradient exists ($\nabla_{\mu} > 0$) and the heavy-component

abundance exceeds the compositional threshold ($x_{\text{heavy}} > x_{\text{inhib}}$). We combine these criteria into a smooth inhibition “strength” $w \in [0, 1]$ that controls how strongly the local temperature gradient is driven toward the radiative (diffusion-limited) gradient:

$$\begin{aligned} w &= w_{\mu} w_{x_{\text{inhib}}}, \\ w_{\mu} &= \mathcal{H}(\nabla_{\mu}), \\ w_{x_{\text{inhib}}} &= \mathcal{H}\left(\frac{x_{\text{heavy}}}{x_{\text{inhib}}}\right), \end{aligned} \quad (27)$$

where $\mathcal{H}$ is a Hill function used to smooth the transition into and out of the inhibited regime and to avoid numerical noise.

The temperature gradient is evaluated by interpolating between a baseline gradient and the radiative required gradient:

$$\nabla = (1 - w) \nabla + w \nabla_{\text{rad}}, \quad (28)$$

where $\nabla = \nabla_{\text{ad}}$ in convective regions and $\nabla = \nabla_{\text{rad}}$ otherwise. In the limit $w \rightarrow 1$, the temperature gradient approaches the diffusion-limited value, while for $w \rightarrow 0$, the solution reverts to the transport regime outside the Ledoux stability region.

Both types of basal boundary layers are global bottlenecks for the intrinsic luminosity. When a Ledoux-stable compositional gradient is present, L_{int} is capped at the maximum luminosity that can be transmitted through the inhibited layer without violating Ledoux stability. This constraint is conceptually analogous to the luminosity limit imposed by the basal thermal boundary layer of thickness δ_{BL} , which bounds the net heat flux delivered from the magma ocean into the envelope. The distinction between the two lies in resolvability: the Ledoux-stable region is sufficiently extended that its stabilizing gradient can be described explicitly, whereas the basal thermal boundary layer is often too thin to resolve and is therefore parameterized as a global flux limit.

We treat heat transport in the nonconnective hydrogen boundary layer as radiative, neglecting conduction. To illustrate why, we compare an effective radiative conductivity $\lambda_{\text{rad}} = 16 \sigma T^3 / (3 \kappa \rho)$ with the thermal conductivity λ_{cond} derived from the nonelectronic molecular λ_{cond} of $\sim 2 \text{ W m}^{-1} \text{ K}^{-1}$ appropriate for our model conditions (M. French et al. 2012). At the conditions that obtain for a typical stable boundary layer ($T \sim 3000 \text{ K}$, $P \sim 5 \text{ GPa}$), $\kappa \approx 20 \text{ cm}^2 \text{ g}^{-1}$ (R. S. Freedman et al. 2014), yielding $\lambda_{\text{rad}} \approx 14 \text{ W m}^{-1} \text{ K}^{-1}$, exceeding $\lambda_{\text{cond}} \sim 2 \text{ W m}^{-1} \text{ K}^{-1}$ by a factor of 7. Because of the marked temperature dependence of the mean opacity, whereby κ rises by more than an order of magnitude between 3000 and 5000 K, faster than T^3 , there is a radiative/conductive crossover near 4300–4900 K, depending on density, above the temperatures in our boundary layers. Accordingly, our result is consistent with W. Misener et al. (2023), who assumed higher temperatures of $\sim 5000 \text{ K}$ at the base of the atmosphere, where $\kappa \gtrsim 600 \text{ cm}^2 \text{ g}^{-1}$, resulting in conduction competing with radiative diffusion. The difference reflects the $\sim 2000 \text{ K}$ difference in base temperature.

3.3. Planet Radius

The planetary radius for these models is defined using the chord optical depth (T. Guillot 2010), coupled with opacities from R. S. Freedman et al. (2014) evaluated at metallicities corresponding to the local gas composition. The binodal interface between the magma ocean and the overlying

envelope constrains the surface temperature at a given pressure to a relatively narrow range, such that the thermal state of the system is expressed primarily through the radial position of the boundary rather than through large variations in temperature. Since the atmospheric entropy and scale height are controlled by the intrinsic luminosity, the basal boundary layer directly governs the achievable planetary radius. In this sense, the core–envelope boundary layers provide upper bounds on the radii of sub-Neptunes.

3.4. Physical Chemistry

As a result of the retention of significant fractions of gravitational potential energy during accretion, sub-Neptunes are expected to have formed hot enough such that a global magma ocean is present beneath their primary hydrogen-rich atmospheres (e.g., E. D. Young et al. 2024). Cooling times moderated by the dense atmospheres are such that the magma oceans have likely persisted over Gyr timescales, meaning that a significant fraction of sub-Neptunes observed today should have magma oceans beneath their envelopes (S. Ginzburg et al. 2016; W. Misener & H. E. Schlichting 2022). Therefore, to understand the chemistry and structure of sub-Neptunes requires an understanding of the interaction between the hydrogen-rich envelopes and melts, or supercritical fluids, comprising the cores of the planets (E. S. Kite et al. 2020; H. E. Schlichting & E. D. Young 2022).

Recent work has applied advances in our understanding of the physical chemistry of H_2 –silicate–Fe metal systems to sub-Neptunes (E. D. Young et al. 2024, 2025; J. G. Rogers et al. 2025b; T. Gilmore & L. Stixrude 2026). That work shows that the interface separating the outer envelope of a sub-Neptune (i.e., its atmosphere, including the supercritical fluid phase, where appropriate) and the underlying magma ocean is a phase boundary that can be well approximated as the binodal (or solvus) surface in the MgSiO_3 – H_2 system (E. D. Young et al. 2024; J. G. Rogers et al. 2025b; T. Gilmore & L. Stixrude 2026). This boundary moves inward toward the center as the planet cools (J. G. Rogers et al. 2025b). Interior to the boundary, the magma ocean is a supercritical mixture of silicate (modeled as MgSiO_3) and hydrogen with or without a distinct Fe-rich metal phase. For planets with sufficient hydrogen that their cores have greater than about 1% by mass H_2 , Fe metal, MgSiO_3 (silicate), and H_2 are entirely miscible (E. D. Young et al. 2025). In these cases, the cores are a single phase throughout, except at the most shallow depths. We emphasize that the binodal phase boundary replaces the notion of a melt–envelope interface imposed at a specified surface. In that picture, the pressure and temperature at the interface are set by the overlying envelope mass, gravity, and envelope adiabat. In our model, by contrast, the phase diagram fixes the pressure–temperature condition for coexistence, and hydrostatic equilibrium determines the radius at which the phase change occurs in the planet.

3.4.1. Core–Envelope Boundary

By determining the common tangents to the Gibbs free energy of mixing in the MgSiO_3 – H_2 system—using the convex hull for the function, for example—the values for the mole fractions of hydrogen, x_{H_2} , for melt and vapor in equilibrium are obtained. These coexisting compositions at various P and T values define the binodal surface that separates

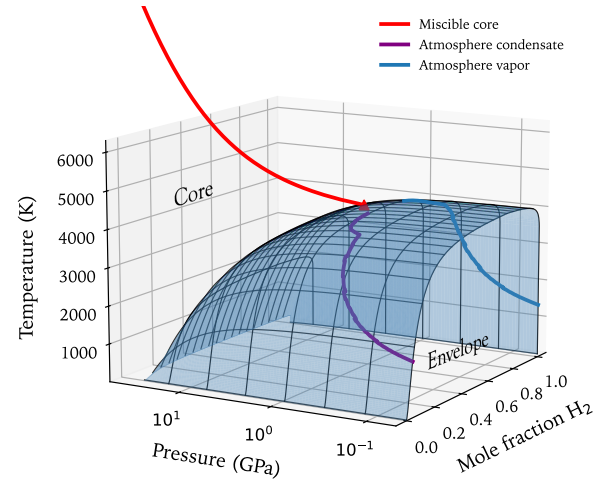


Figure 1. The binodal surface separating the core and envelope for a $6 M_{\oplus}$, 3 wt% H_2 sub-Neptune with a fully miscible core prior to Gyr timescales of cooling. The paths through the one-phase core and through the two-phase envelope are shown.

the cores from overlying envelopes. The nonideal mixing between MgSiO_3 and H_2 calculated by T. Gilmore & L. Stixrude (2026) is used here. Their density functional theory molecular dynamics (DFT-MD) simulations yield the binary free energy of mixing expression

$$\begin{aligned} \Delta \hat{G}_{\text{binary mix}} = & (L_{\text{CB}}x_{\text{H}_2} + L_{\text{BC}}(1 - x_{\text{H}_2}))x_{\text{H}_2}(1 - x_{\text{H}_2}) \\ & \times \left(1 - \frac{T}{\tau} + \frac{P}{\pi}\right) + RT(x_{\text{H}_2} \ln x_{\text{H}_2} \\ & + (1 - x_{\text{H}_2}) \ln(1 - x_{\text{H}_2})), \end{aligned} \quad (29)$$

where L_{CB} and L_{BC} are the binary interaction parameters, and x_{H_2} is the mole fraction of H_2 along the binary join. The values for the subregular mixing parameters that fit the DFT-MD simulations are: $L_{\text{CB}} = 622,000 \text{ J mol}^{-1}$, $L_{\text{BC}} = -4950 \text{ J mol}^{-1}$, $\tau = 4800 \text{ K}$, and $\pi = -35 \text{ GPa}$. T. Gilmore & L. Stixrude (2026) find that the addition of minor elements like Al has minimal effect on the position of the binodal surface in P – T – x_{H_2} space.

Using reasonable estimates for the initial specific entropies and associated luminosities, J. G. Rogers et al. (2025b) showed that the boundary between the core and envelope for younger sub-Neptunes (ages of 10^7 yr) is predicted to be near 1 GPa and 4000 K for a $6 M_{\oplus}$ sub-Neptune with 3wt% total hydrogen. With cooling over a Gyr, the boundary shifts inward and is characterized by higher pressures and lower temperatures in the vicinity of 4 GPa and 3000 K. The specific conditions at the binodal surface depend on the bulk H_2 composition of the planet as well as its mass and equilibrium temperature. For comparison, and using the topology from E. D. Young et al. (2024) and J. G. Rogers et al. (2025b), the binodal surfaces for the example of a $6 M_{\oplus}$, 3% H_2 fully miscible planet for the early and late cases based on the current modeling are shown in Figures 1 and 2.

Note that the piercing point of the path through the core (red curve) that defines the magma ocean–envelope interface occurs at higher P and lower T for the older planet in Figure 2. Calculations such as these illustrate that the pressure of the binodal is a surrogate for the relative age of a sub-Neptune, all else equal. The binodal separating the core from the envelope

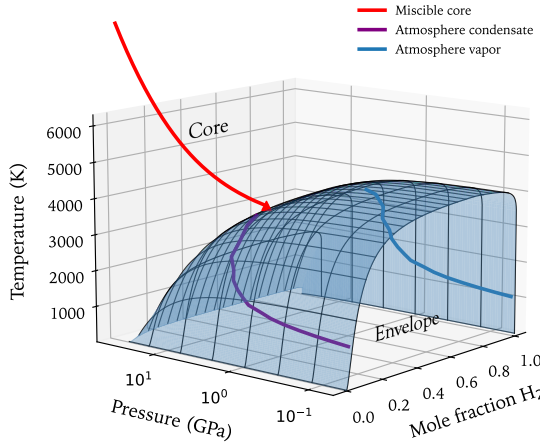


Figure 2. The binodal surface separating the core and envelope for a $6 M_{\oplus}$, 3 wt% H_2 sub-Neptune with a fully miscible core after approximately a Gyr time interval of cooling. The paths through the one-phase core and through the two-phase envelope are shown.

spans a significantly narrower range of conditions than the bases of atmospheres in “classical” models where there is no interaction between the hydrogen in the envelope and the magma ocean core (E. D. Young et al. 2024; J. G. Rogers et al. 2025b).

3.4.2. Core Miscibility

In order to determine the state of the core, given the planet’s pressure–temperature structure and its mass fraction of hydrogen, we make use of the ternary phase equilibria from E. D. Young et al. (2025). With this approach, we construct a thermodynamically consistent ternary mixing model for miscible $MgSiO_3$ –Fe– H_2 melts by extrapolating three independently constrained binary systems ($MgSiO_3$ – H_2 , $MgSiO_3$ –Fe, and Fe– H_2) into ternary composition space. The phase stability is determined by minimizing the molar Gibbs free energy of mixing at fixed pressure and temperature. The total free energy of mixing is written as the sum of an ideal configurational entropy term and a nonideal (excess) mixing term. The ideal contribution is given by $RT \sum_i x_i \ln x_i$, where x_i are the mole fractions of Fe (A), H_2 (B), and $MgSiO_3$ (C). Nonideal interactions are parameterized using a subregular solution formalism, with the interaction parameters taken directly from published DFT-MD simulations and experimental constraints on the three binary joins.

The resulting Gibbs free energy of mixing for the ternary system is obtained following the Muggianu–Jacob projection method:

$$\begin{aligned} \Delta \hat{G}_{\text{mix}} = & RT(x_A \ln x_A + x_B \ln x_B + x_C \ln x_C) \\ & + (1/2)x_A x_B (L_{AB}(1 + x_B - x_A) \\ & + L_{BA}(1 + x_A - x_B)) \\ & + (1/2)x_B x_C (L_{BC}(1 + x_C - x_B) \\ & + L_{CB}(1 + x_B - x_C)) \\ & (1 - T/\tau + P/\pi) \\ & + (1/2)x_C x_A (L_{CA}(1 + x_A - x_C) \\ & + L_{AC}(1 + x_C - x_A)), \end{aligned} \quad (30)$$

where x_A , x_B , and x_C denote the mole fractions of Fe, H_2 , and $MgSiO_3$, respectively, with $x_A + x_B + x_C = 1$. The $MgSiO_3$ – H_2 interaction parameters are $L_{CB} = 6.22 \times 10^5 \text{ J mol}^{-1}$ and

$L_{BC} = -4.95 \times 10^3 \text{ J mol}^{-1}$, with the temperature and pressure dependence governed by $\tau = 4800 \text{ K}$ and $\pi = -35 \text{ GPa}$, as fitted by T. Gilmore & L. Stixrude (2026). The $MgSiO_3$ –Fe interaction is treated as a regular solution with $L_{AC} = L_{CA} = 2.4 \times 10^5 - 28T + 1116P \text{ (J mol}^{-1}\text{)}$, based on DFT-MD calculations for MgO –Fe mixing. The Fe– H_2 interaction parameters are $L_{AB} = 1.38 \times 10^5 - 9500P$ and $L_{BA} = 1.7 \times 10^4 - 9500P \text{ (J mol}^{-1}\text{)}$, calibrated to reproduce experimental and ab initio constraints on the hydrogen solubility in molten iron. No explicit, and unknown, ternary interaction parameter is included ($L_{ABC} = 0$), as is often the case in such applications.

With this free energy of mixing, we can map the regions of miscibility and immiscibility between $MgSiO_3$, Fe, and H_2 as a function of pressure and temperature (a Python code for these calculations, `G_planet_intersection`, can be found online).²

3.4.3. Material Properties of the Core

For liquid silicate densities, we use the approach from E. D. Young et al. (2025). Briefly, for the silicate melt, we use an EoS fit to the $MgSiO_3$ liquid properties determined by N. de Koker & L. Stixrude (2009) using the algorithms of A. S. Wolf & D. J. Bower (2018). The total pressure is computed by combining the elastic (cold) compression term from a Vinet EoS with thermal pressure contributions, following the formulation of A. S. Wolf & D. J. Bower (2018). For liquid iron, we use a Vinet EoS modified to include thermal pressure from Y. Kuwayama et al. (2020). The Grüneisen parameter is a simple function of the compression ratio, which in turn determines the thermal contribution to pressure.

In our models here, we assume that the addition of H and, in some cases, Fe to the supercritical magma ocean changes its density, but does not significantly affect the bulk moduli and their pressure derivatives (we will investigate this assumption in future publications). For regions of the planet inside the binodal, hydrogen and $MgSiO_3$ melt are completely miscible. Based on DFT-MD simulations (E. D. Young et al. 2025), the presence of hydrogen in the silicate melt reduces its density such that a linear mixture of the compressed densities of $MgSiO_3$ and H_2 by volume reproduces the density of the mixture at the conditions of the binodal. We therefore include the effects of H_2 on the density of the supercritical melt by calculating the density of the mixture at the pressures and temperatures of the surface of the magma ocean, ρ_0 . The density of the mixture is

$$\frac{1}{\hat{V}_{\text{mix}}} = x_{H_2} \frac{1}{\hat{v}_{H_2}} + x_{\text{sil}} \frac{1}{\hat{v}_{\text{sil}}}, \quad (31)$$

where $\hat{v}_{H_2}$ and $\hat{v}_{\text{sil}}$ are the molar volumes, and where x_{H_2} and x_{sil} are the mole fractions of hydrogen and silicate in the binary mixture. The densities are obtained from $MW_{\text{mix}}/\hat{V}_{\text{mix}}$, where MW_{mix} is the molecular weight of the mixture. The molar volumes are obtained from the densities and EoSs using ρ_i/MW_i . We fix the densities to be 0.09 g cm^{-3} and 2.5 g cm^{-3} for H_2 and silicate, respectively, at the surface of the magma ocean. These values come from the EoSs for hydrogen

² https://github.com/eyounguc/planet_intersection

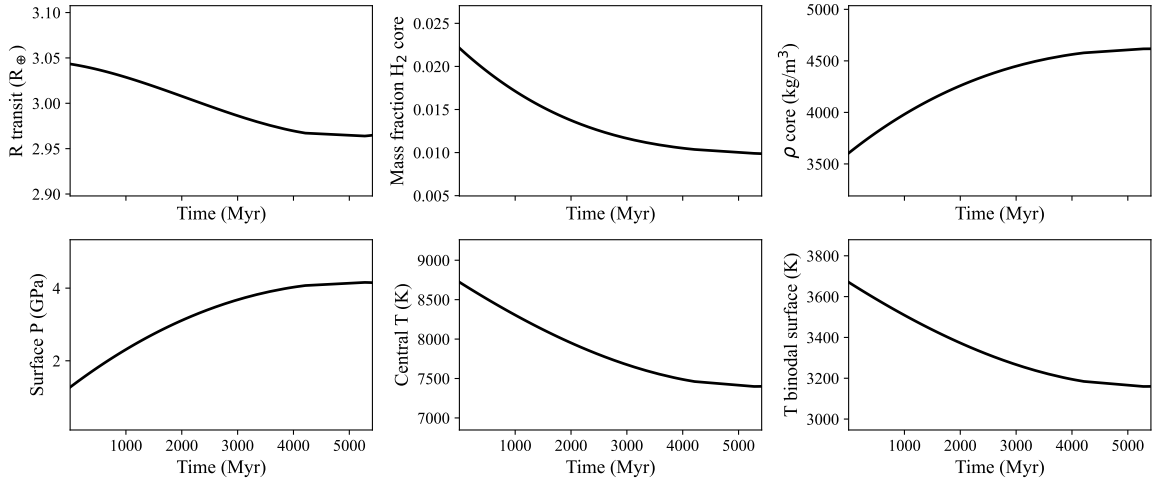


Figure 3. Time evolution of a model sub-Neptune with a total mass of $6M_{\oplus}$ and mass fractions of H_2 of 3%. In this example, $Ra/Ra_c = \times 10^{12}$, resulting in an initial intrinsic luminosity of 6.3×10^{15} W that decreases over the approximately 5 Gyr of evolution to 1.1×10^{15} W.

from G. Chabrier et al. (2019) and ab initio molecular dynamics simulations of $MgSiO_3$ melt at 6000 K and 3.5 GPa (E. D. Young et al. 2025).

Following E. D. Young et al. (2025), we model Fe dissolved in a supercritical silicate- H_2 melt by assigning Fe a partial molar volume that reflects the Fe occupation of Mg-like cation sites within the silicate melt framework. The Fe-site partial molar volume is

$$\bar{V}_{\text{Fe site}}^{\text{eff}} = \phi \propto \bar{V}_{\text{MgSiO}_3}^{\text{pure}}(P, T), \quad (32)$$

and the corresponding effective partial Fe density is

$$\rho_{\text{Fe site}}^{\text{eff}} = \frac{MW_{\text{Fe}}}{\bar{V}_{\text{Fe site}}^{\text{eff}}}, \quad (33)$$

where ϕ denotes the fraction of the pure $MgSiO_3$ formula-unit molar volume attributable to the Mg site, $\alpha = (r_{\text{Fe}}/r_{\text{Mg}})^3$ scales the site volume by the ratio of octahedrally coordinated ionic radii (R. D. Shannon 1976), and $\bar{V}_{\text{MgSiO}_3}^{\text{pure}} = MW_{\text{MgSiO}_3}/\rho_{\text{MgSiO}_3}^{\text{pure}}$ is the molar volume of pure $MgSiO_3$ at the local P and T . Assuming the additivity of specific volumes (no excess mixing volume), the density of the miscible melt with Fe mass fraction w_{Fe} is given by

$$\frac{1}{\rho_{\text{mix}}} = \frac{1 - w_{\text{Fe}}}{\rho_{\text{MgSiO}_3 + H_2}} + \frac{w_{\text{Fe}}}{\rho_{\text{Fe}}^{\text{eff}}}, \quad (34)$$

where $\rho_{\text{MgSiO}_3 + H_2}$ is the density of the $MgSiO_3 + H_2$ mixture derived at P and T . An empirical value for ϕ of 0.5 is obtained from the MgO-Fe system (L. Insixiangmay & L. Stixrude 2025).

3.5. Evolution with Time

We do not use determinative evolutionary paths in this population study, due to their computational cost, as described above. Instead, we use the pressures of the binodal interface between the sub-Neptune supercritical magma oceans and envelopes as a means of specifying the relative ages of our model planets. Luminosity could also be used (J. G. Rogers et al. 2025b), but the initial luminosities are not a free parameter in our inverse statistical approach. An example evolutionary path is shown in Figure 3. The quasi-static

evolution curves are calculated by specifying the planet mass M_p , bulk hydrogen mass fraction, equilibrium temperature T_{eq} , and an initial binodal pressure of 0.9 GPa. The binodal pressure was increased incrementally. For each step, we calculated the total energy of the planet E_p and the associated intrinsic luminosity L_{int} . The time step associated with a step from P_i to P_{i+1} was obtained from the energy balance

$$dt = \frac{|E_p(P_{i+1}) - E_p(P_i)|}{L_{\text{int}}}, \quad (35)$$

where L_{int} comes from Equation (14). The total energy E_p is calculated from

$$E_p = \int_0^{R_p} \left(-\frac{GM(r)}{r} + c T(r) \right) \rho(r) 4\pi r^2 dr, \quad (36)$$

where c is the specific heat capacity for the melt or envelope using the EoSs and species described above. The pressure steps are sufficiently small to ensure that dt is approximately 3 to 4 orders of magnitude smaller than the cooling-time constant $\tau_{\text{cool}} = |E_p|/L_{\text{int}}$.

The example evolution track shown in Figure 3 illustrates several salient features relevant to this study. Most importantly, the shifts in radius are relatively small over the timescales of interest defined by the ages of the host stars for sub-Neptunes derived from the planet archive (Appendix A). These ages define a broad log-normal-like distribution with a mode near 2.5 Gyr and a median of 4.3 Gyr. With cooling, the planet's silicate-hydrogen binodal surface moves inward with time (J. G. Rogers et al. 2025b). As the binodal moves inward, the pressure at the surface increases from <2 GPa to about 4 GPa and thereafter changes extremely slowly. The timescale for evolution to nearly steady state is of order a few Gyr, where the luminosity is governed by the basal radiative boundary, or hundreds of millions of years, if this layer were not to be present. The shape of the binodal phase boundary that separates the supercritical magma ocean from the overlying hydrogen-rich envelope dictates the finite range of surface pressures. At lower pressures, the temperature of the interface is higher, but the critical temperature (at the crest of the binodal, the maximum for a given pressure) approaches a maximum of 4350 K at 0 GPa (about 100 K higher than at 1 GPa). Conversely, as the pressure exceeds 4 GPa, and the

binodal temperature approaches 2000 K for the hydrogen mass fractions of interest here (a few percent or less by mass), no hydrogen would remain in the supercritical magma ocean, and the binodal ceases to be relevant. The evolution curves also illustrate that hydrogen is continually exsolved from the supercritical magma ocean into the envelope, and this process slows considerably with time, as shown previously by J. G. Rogers et al. (2025b). Less massive envelopes (lower initial pressures at the binodal interface) result in greater intrinsic luminosities and faster evolution than shown in this figure.

3.6. Atmospheric Escape Model

3.6.1. Energy-limited Escape

For each model planet, the likelihood that the envelope (atmosphere) survived for the time interval corresponding to the planet's assigned age is evaluated in the population study. This is the probabilistic approach to determining which planets have extant envelopes and which should be relegated to stripped cores. We use an energy-limited prescription for atmospheric loss by irradiation treated as a continuous mass-loss process. The time-integrated mass loss determines whether an atmosphere is retained or stripped. We augment this by a simple criterion for hydrodynamic escape.

Atmospheric loss is modeled using an energy-limited prescription based on the atmospheric mass fraction:

$$x(t) \equiv \frac{M_{\text{atm}}(t)}{M_p(t)}, \quad (37)$$

where the total planet mass is

$$M_p(t) = M_{\text{core}} + M_{\text{atm}}(t) \quad (38)$$

and the core mass M_{core} is fixed. In this case, atmospheric escape is driven by extreme-ultraviolet stellar irradiation (XUV). Because mass is removed from both the atmosphere and the total planet mass, it follows that the evolution of x is

$$\frac{dx}{dt} = -\frac{\dot{M}_{\text{esc}}}{M_p} (1 - x). \quad (39)$$

The instantaneous energy-limited atmospheric mass-loss rate is

$$\dot{M}_{\text{esc}} = \eta \frac{\pi R_{\text{XUV}}^3 F_{\text{XUV}}(t)}{G M_p K}, \quad (40)$$

where η is the heating efficiency, F_{XUV} is the stellar XUV flux at the planet, and $K \leq 1$ accounts for the Roche-lobe effects of the reduction in binding energy where the atmosphere extends to a fraction of the planet's Hill radius (M. Salz et al. 2016). The stellar XUV luminosity is parameterized by an initial saturated phase followed by power-law decay (e.g., J. G. Rogers & J. E. Owen 2021):

$$L_{\text{XUV}}(t) = \begin{cases} \varepsilon_{\text{XUV}} L_*, & t \leq t_{\text{sat}}, \\ \varepsilon_{\text{XUV}} L_* \left(\frac{t}{t_{\text{sat}}}\right)^{-\alpha}, & t > t_{\text{sat}}, \end{cases} \quad (41)$$

with the associated flux being

$$F_{\text{XUV}}(t) = \frac{L_{\text{XUV}}(t)}{4\pi a^2}. \quad (42)$$

The Roche geometric factor is taken to be (M. Salz et al. 2016)

$$K = 1 - \frac{3}{2\xi} + \frac{1}{2\xi^3}, \quad (43)$$

$$\xi \equiv \frac{R_L}{R_{\text{XUV}}}, \quad (44)$$

with

$$R_L \simeq a \left(\frac{M_p}{3M_*} \right)^{1/3}, \quad (45)$$

where M_* is the stellar mass and a is the planet's semimajor axis.

The XUV absorption radius is parameterized as

$$R_{\text{XUV}} = R_p + n_H H, \quad (46)$$

where the scale height is given by

$$H = \frac{k_B T_{\text{therm}}}{\mu} \frac{R_p^2}{GM_p}. \quad (47)$$

Here, R_p is the photospheric radius, T_{therm} is a characteristic upper-atmosphere temperature, and μ is the mean molecular mass. The parameter n_H is treated as an order-unity factor encapsulating thermospheric structure and opacity rather than as a literal number of physical scale heights. It is a simplification (see M. Salz et al. 2016) that is justified insofar as our results are not critically dependent on this value within plausible limits.

The characteristic upper-atmosphere temperature T_{therm} is obtained from the stellar properties by scaling it to the equilibrium temperature implied by those properties:

$$T_{\text{eq}} = \left[\frac{(1 - A) L_*}{16\pi\sigma_{\text{SB}} a^2 f} \right]^{1/4}, \quad (48)$$

where A is the Bond albedo, σ_{SB} is the Stefan–Boltzmann constant, and f accounts for heat redistribution (with $f = 1$ for full-planet reradiation). The upper-atmosphere temperature is then taken to be

$$T_{\text{therm}} = \alpha_T T_{\text{eq}}, \quad (49)$$

where $\alpha_T \gtrsim 1$ is a dimensionless scaling factor that captures the prospects for additional thermospheric heating by XUV irradiation. In practice, α_T is treated as a fixed parameter. Varying α_T primarily rescales the default atmospheric scale heights and hence the effective XUV absorption radii, and therefore plays a role analogous to other escape-efficiency parameters, such as η and n_H .

Combining these elements, the evolution equation solved numerically is

$$\frac{dx}{dt} = -(1 - x) \frac{\eta \pi R_{\text{XUV}}^3}{G M_p K} \frac{L_{\text{XUV}}(t)}{4\pi a^2}, \quad (50)$$

Table 1
Atmospheric Escape Probability Parameters Used in Models

Parameter	Definition	Adopted Value
η	Heating efficiency	0.16
ε_{XUV}	Saturated XUV fraction	10^{-3}
t_{sat}	XUV saturation time	100 Myr
α	Post-saturation decay index	1.1
n_H	XUV absorption scale factor	2.0
μ	Mean molecular mass	$2.3 m_H$
T_{therm}	Upper-atmosphere temperature	$3 T_{\text{eq}}$
x_{thresh}	Stripping threshold	10^{-4}
σ	Probability transition width	10 Myr
ε	Survival probability at t_{strip}	10^{-3}
M_*	Stellar mass	$0.9 M_{\odot}$
L_*	Stellar luminosity	$1.0 L_{\odot}$

with

$$M_p = \frac{M_{\text{core}}}{1 - x}. \quad (51)$$

Example calculations based on the parameters in Table 1, in part from M. Salz et al. (2016) and J. G. Rogers & J. E. Owen (2021), illustrate the sharp nature of the energy-limited escape (Figure 4). Planets either retain their atmospheres or they do not, with the time interval for partial escape being relatively limited for the timescales of order a Gyr of interest here. Therefore, escape can be treated as a probability for atmospheric “survival.”

3.6.2. Survival Probability

The atmospheric evolution is integrated forward in time using explicit time stepping:

$$M_{\text{atm}}(t + \Delta t) = M_{\text{atm}}(t) - \dot{M}_{\text{esc}}(t) \Delta t, \quad (52)$$

and therefore

$$x(t + \Delta t) = \frac{M_{\text{atm}}(t + \Delta t)}{M_{\text{core}} + M_{\text{atm}}(t + \Delta t)}. \quad (53)$$

The characteristic stripping time t_{strip} is defined as the first time $x(t)$ falls below a threshold x_{thresh} that is near zero.

The default probability mapping converts t_{strip} into a smooth survival probability:

$$p_{\text{survive}}(\tau) = \frac{1}{2} \operatorname{erfc}\left(\frac{\tau - t_{\text{strip}} + \Delta}{\sqrt{2} \sigma}\right), \quad (54)$$

with a temporal offset

$$\Delta = \sqrt{2} \sigma \operatorname{erfc}^{-1}(2\varepsilon), \quad (55)$$

such that $p_{\text{survive}}(\tau = t_{\text{strip}}) = \varepsilon$.

3.6.3. Hydrodynamic Escape/Core-powered Mass Loss

The energy-limited photoevaporation atmosphere escape calculations can in some cases result in an atmosphere that is unstable against hydrodynamic escape (Parker wind/Bondi escape). To account for this, we apply a threshold parameter that accounts for hydrodynamic escape in the form of the escape parameter $\lambda_{\text{rcb}} = GM_p \mu / (k_B T_{\text{rcb}} R_{\text{rcb}})$, where μ is the mean molecular mass and T_{rcb} is the temperature, each evaluated at the radial position of the radiative-convective boundary (RCB), R_{rcb} . This serves to include core-powered

mass loss implicitly (S. Ginzburg et al. 2016, 2018; A. Gupta & H. E. Schlichting 2019), where the planet’s intrinsic cooling luminosity maintains an elevated T_{rcb} for Gyr timescales, keeping λ_{rcb} small and the atmosphere vulnerable to escape. In our models, T_{rcb} comes in part from the planet’s intrinsic luminosity. The λ_{rcb} threshold therefore captures the essence of core-powered mass loss without requiring an explicit integration of the coupled thermal and mass-loss evolution. It operates as a safeguard beyond XUV escape alone that guards against implausible atmospheres.

We evaluate the threshold value for λ_{rcb} based on the analysis of A. Gupta & H. E. Schlichting (2019). The mass-loss rate due to a Parker wind at the sonic point is

$$\dot{M} = \pi G^2 M_p^2 P_{\text{rcb}} \frac{\mu^{5/2}}{(k_B T_{\text{rcb}})^{5/2}} e^{-\lambda_{\text{rcb}}}, \quad (56)$$

where P_{rcb} is the pressure at the RCB and, again, λ_{rcb} is the escape parameter evaluated at the RCB. Equation (56) is obtained by substituting the sonic-point radius $R_s = GM_p / (2c_s^2)$ and the ideal-gas relation $\rho_{\text{rcb}} = P_{\text{rcb}} \mu / (k_B T_{\text{rcb}})$ into the equation for Bondi escape. The corresponding atmospheric loss timescale is $t_{\text{loss}} = x_{\text{atm}} M_p / \dot{M}$, where $x_{\text{atm}} \equiv M_{\text{atm}} / M_p$ is the mass fraction of the envelope (atmosphere). Based on this definition for the timescale of atmosphere loss, we have

$$t_{\text{loss}} = \frac{x_{\text{atm}} (k_B T_{\text{rcb}})^{5/2}}{\pi G^2 M_p P_{\text{rcb}} \mu^{5/2}} e^{+\lambda_{\text{rcb}}}. \quad (57)$$

We require that the atmosphere survive at least one protoplanetary disk dispersal timescale, $t_{\text{loss}} \geq t_{\text{disk}}$, and solve for λ_{rcb} based on that condition:

$$\lambda_{\text{rcb}} \geq \ln\left(\frac{t_{\text{disk}} \pi G^2 M_p P_{\text{rcb}} \mu^{5/2}}{x_{\text{atm}} (k_B T_{\text{rcb}})^{5/2}}\right). \quad (58)$$

Evaluating Equation (58) at representative values ($t_{\text{disk}} = 3$ Myr, $M_p = 1\text{--}20 M_{\oplus}$, $x = 10^{-4}\text{--}10^{-1}$, $P_{\text{rcb}} = 10$ bar, $T_{\text{rcb}} = 1020$ K, and $\mu = 2.3 m_H$) yields $\lambda_{\text{min}} \approx 22\text{--}32$, with an intermediate value of ≈ 26 at $M_p = 5 M_{\oplus}$ and $x_{\text{atm}} = 0.01$. Because λ_{rcb} depends on the logarithm of the various terms, the threshold is relatively insensitive to the precise values of the planet parameters: varying t_{disk} over 1–10 Myr, M_p over 1–20 $M_{\oplus}$, x over four decades, and T_{rcb} over 300–2000 K shifts λ_{min} by at most a few units in each case. We therefore adopt $\lambda_{\text{rcb}} = 20$ as a conservative threshold, noting that no combination of physically plausible parameters produces λ_{min} below about 22. Planets with escape parameters < 20 at the RCB have their atmospheres stripped in the model population.

4. Monte Carlo Sampling

A synthetic population of planets based on the modeling described above (contained in the Python code `Planet-LAB`)³ was obtained using a Monte Carlo sampling procedure. Here, the stochastic selection of input parameters and the probabilistic assignment of observable states is described.

³ https://github.com/eyoung UCLA/Planet_Lab

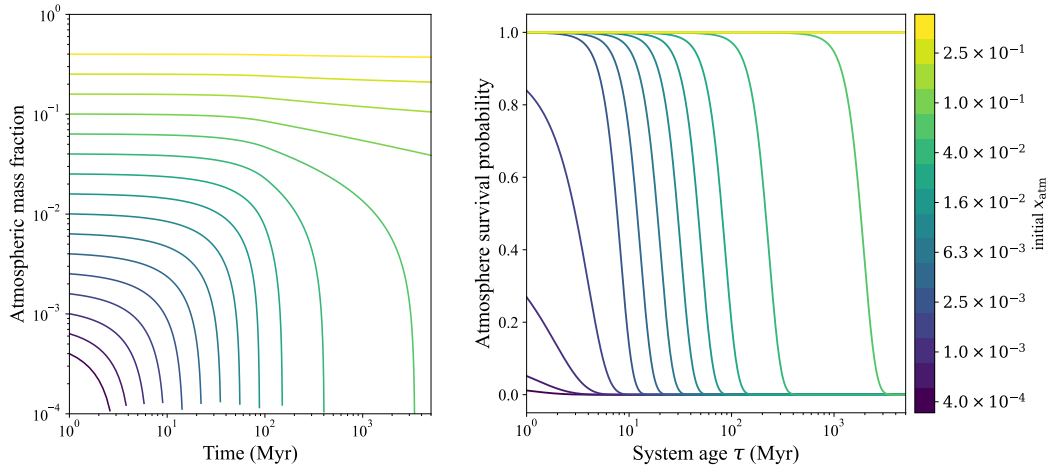


Figure 4. Atmosphere mass fractions as a function of time for planets each with a total mass of $5M_{\oplus}$ and a period of 10 days. The panel on the left shows contours for the different initial atmosphere mass fractions. The panel on the right shows the associated atmosphere survival probabilities. The color bar for the initial mass fractions of atmosphere applies to both plots.

4.1. Primary Parameter Sampling

A total of 1012 planetary realizations were generated for the results reported here. The results are not sensitive to the precise number beyond ~ 200 draws. Each realization is prescribed by the total planet mass (M_p), total weight percent H_2 , pressure at the binodal surface, and equilibrium temperature. While we assign distinct equilibrium temperatures for the atmospheric loss calculations (Section 3.6), we choose to use a single equilibrium temperature, $T_{eq} = 500$ K, for constructing the pre-escape planet models. This value is taken to be representative for the range in periods of about 10–100 days. In reality, T_{eq} can vary from about 1000 to 400 K for a solar-mass star over this range in period, depending upon the albedo. Applying individual T_{eq} values over this range in period has only negligible effects on our results.

Planet masses are drawn from a log-uniform distribution:

$$1.7 \leq M_p/M_{\oplus} \leq 12, \quad (59)$$

in order to obtain equal probability per decade in mass.

The bulk hydrogen mass fraction prior to atmospheric loss is sampled from a truncated log-normal distribution. This distribution type is motivated by expectations from agglomerations of protoplanets with hydrogen envelopes coupled with fractional losses of hydrogen during multiple accretion events (R. Gibrat 1931); a log-normal distribution is an emergent property of stochastic assembly (Appendix B). The adopted distribution has a median value of 1.8 wt% and a logarithmic dispersion of 0.4 dex, with fixed bounds of 0.05 and 10 wt%, to exclude unrealistically gas-poor or gas-rich cases. The physical implications of the distribution parameters are discussed in Appendix B.

The binodal surface pressure P_s is drawn from a uniform distribution between 0.2 and 5 GPa for planets with $M_p < 3M_{\oplus}$ and between 0.4 and 5 GPa for more massive planets. This weak mass dependence is the result of numerical considerations, and the results are not sensitive to the precise lower bounds, as described above. In practice, for almost all realizations, the upper bound of 5 GPa, where binodal temperatures are low, is never attained, as Planet_LAB reduces the pressure if the total amount of H_2 is insufficient to avoid negative concentrations of hydrogen in the cores. The practical upper limit is 4 GPa (e.g., Figure 3).

4.2. Metal Core Classification

Each model planet realization is assigned a binary metal core flag prior to forward modeling, in order to decide whether the forthcoming model should include a separate Fe-rich metal core or not (i.e., whether the planet is differentiated in the terrestrial sense). The classification predictor was obtained by analysis of 218 random planet realizations that correlated the input variables with the phase equilibria described in Section 3.4.2 and Section 3.4.3. Each of these random draws produced planets with a weight fraction of H_2 in the core, and one can show by comparisons with the ternary phase equilibria that for core hydrogen concentrations $< 0.5\%$, a discrete metal phase is expected to form with a bulk density greater than the surrounding silicate, indicating a strong likelihood for metal/silicate differentiation. A logistic function was used as a classification predictor for whether a given set of model parameters leads to metal/silicate differentiation and the formation of a metal core:

$$p(\text{metal core}) = \frac{1}{1 + e^{-z}}, \quad (60)$$

where z is a linear predictor formed from the input parameters:

$$z = 5.4069 - 0.5040 (M_p/M_{\oplus}) - 3.7032 (H_2/\text{wt}\%) + 1.0793 (P_s/\text{GPa}). \quad (61)$$

The coefficients in Equation (61) were determined by logistic regression applied to 218 exploratory model realizations using the Python statsmodels.api routine `Logit`, where the outcome variable was binary (a metal core was present or absent). Here,

$$z = \ln\left(\frac{p}{1-p}\right), \quad (62)$$

so that $z = 0$ corresponds to $p = 0.5$. The regression therefore determines the hyperplane in parameter space at which the probability of forming a metal core is 50%. The resulting predictor reproduces the classifications of the training models with an accuracy better than 92%.

Planets with $z > 0$ are classified as metal core objects, and the models are run with discrete metal cores using an Earthlike metal fraction of 0.33. Based on prior random model

realizations, an empirical mapping between the total hydrogen and density deficit of the metal phase comprising the differentiated metal core is invoked for each model where differentiation is indicated. The linear mapping is $\Delta\rho_{\text{metal}} = 0.315$ (planet H_2 weight percent), motivated by the ternary phase-equilibria trends explored in our preliminary calculations. We have, therefore, $\rho_{\text{metal}} = \rho_{\text{liquid Fe}}(1 - 0.315 \text{ (planet } \text{H}_2 \text{ weight percent)})$ for our differentiated models.

4.3. Assignment of Evolutionary Properties

After producing a model planet, a second stochastic step assigns evolutionary and orbital properties intended to represent an observational population. For each planet realization, a stellar age τ , orbital period P_{orb} , and initial atmospheric mass fraction parameter x_0 are sampled independently and assigned to the planet.

System ages are drawn from a kernel density estimate (KDE) constructed from stellar ages in the comparison sample extracted from the NASA Exoplanet Archive. Sampling is restricted to the interval

$$50 \leq \tau \leq 10^4 \text{ Myr}, \quad (63)$$

consistent with the empirical age distribution of the observed systems. Orbital periods are sampled from a log-uniform distribution between 7 and 100 days, consistent with the period range considered in the observational comparison.

The initial atmospheric mass fraction x_0 , defined as the ratio of atmospheric mass to total planet mass at formation, is treated as a stochastic initial condition. Because the primordial envelope mass could plausibly correlate with the total hydrogen inventory of a planet, we also experimented with introducing a correlation between x_0 and the bulk H_2 mass fraction prior to atmospheric loss. Tests with correlation coefficients as large as $\rho = 0.9$ produced results that were statistically indistinguishable from those obtained with independent draws. We therefore adopt the simpler assumption of independent sampling in the models reported here.

Values of x_0 are drawn from a log-normal distribution with a median of 2×10^{-2} (a trial value consistent with the median total hydrogen abundance) and a multiplicative 1σ scatter of 2, anticipating order-of-magnitude variability in the primordial envelope masses. The distribution is truncated to the interval 10^{-5} – 10^{-1} to exclude unphysical values in this context. For all planet realizations, the initial atmospheric mass fraction must satisfy $x_0 \geq x_{\text{pd}}$, where x_{pd} is the modeled present-day atmospheric mass fraction.

4.4. Atmospheric Survival as a Bernoulli Process

Atmospheric retention is treated probabilistically. For each realization, the atmosphere survival probability $p_{\text{survive}}(\tau)$ is computed from the sampled parameters and planetary properties using the atmospheric escape framework described in Section 3.6.2. A Bernoulli trial is then performed, such that planets retain their atmospheres with probability p_{surv} and are otherwise represented by their stripped cores.

An additional stability criterion is imposed such that models with $\lambda_{\text{rcb}} < 20$ are always classified as stripped, corresponding to instances where envelopes are dynamically unstable (see Section 3.6.3).

4.5. Resulting Population

The Monte Carlo realizations comprise an ensemble of model planets conditioned on physically motivated priors and filtered by probabilistic atmospheric survival, enabling direct comparison with observed mass–radius and period distributions.

5. Results

5.1. Model Planets

Two examples of the interiors of model planets are shown in Figures 5 and 6, representing early and late stages of evolution, respectively. The figures show pressure versus depth in the molten interior, and three phase diagrams at three representative depths. In both cases, the planets are $6M_{\oplus}$ with 3wt% total hydrogen. Figure 5 shows the state early, with a binodal pressure, P_s , of 0.3 GPa, while Figure 6 shows a later stage of evolution, where L_{int} has decreased from the initial value of 3.3×10^{16} W in Figure 5 to 1.6×10^{15} W and the binodal pressure has increased accordingly to 4 GPa.

At this total hydrogen mass fraction, the core of this planet is predicted to be essentially one phase throughout its history. Inspection of the core compositions in ternary space in the two figures shows that exsolution of H_2 from the core with cooling lowers the core H_2 fraction, placing it on the edge of the two-phase field. However, application of the lever rule indicates that the core is almost entirely composed of the phase at the high-hydrogen end of the tie line passing through the core composition. This phase is nothing like an Fe core in the terrestrial sense, but instead is composed of 0.3 MgSiO_3 , 0.35 Fe, and 0.35 H_2 by mole, or 60% MgSiO_3 , 38.5% Fe, and 1.5% H_2 by mass (based on simple conversion using molecular weights). This is an example of an undifferentiated planet in our model architecture.

Also shown in Figures 5 and 6 is bulk Earth with 0.17wt% H_2 (e.g., E. D. Young et al. 2023), showing the coexistence of a silicate (Earth’s mantle) and an H-bearing Fe metal phase (Earth’s metal core). Planets between the 3% for these sub-Neptune models and Earth will have core hydrogen concentrations intermediate between these endmembers, with 0.5% by mass being the cutoff for silicate/metal differentiation, as prescribed by the slopes of the tie lines between the silicate and metal phases and implemented using Equation (61).

5.2. Reference Planets in Mass–Radius Space

The reference mass versus radius diagram for the 261 archive planets in this study that result from filtering for periods of less than 100 days and mass precision of $\leq 30\%$ is shown in Figure 7. The contours are for the equal kernel density of points. No points are shown below the pure Earthlike mass versus radius curve, since these planets are not included in our models. A curve for 50% by mass water ice, based on an Earthlike core mixed with low-pressure water ice (see A. Aguichine et al. 2021), is shown for reference.

The different types of planets in this population are exemplified by the seven specific examples shown in Figure 7. We can use these planets to define roughly four types of super-Earths and sub-Neptunes for comparisons with our models. Planet 55 Cancri e is near the super-Earth/sub-Neptune boundary with an orbital period of about 1 day and an equilibrium temperature of 2400 K. It is almost certainly a lava

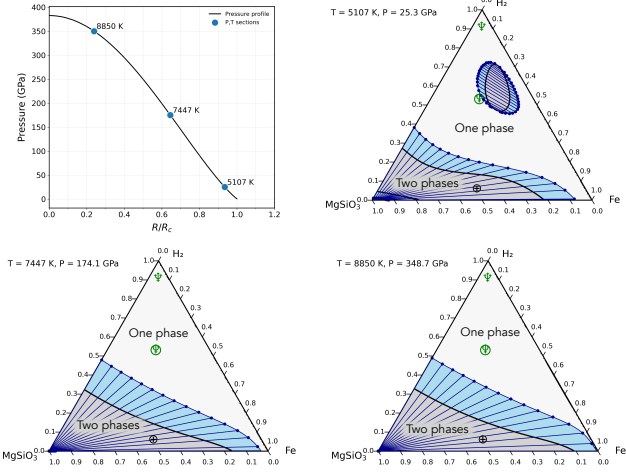


Figure 5. Phase diagrams for the interior of a model $6M_{\oplus}$ planet that has 3wt% total hydrogen with a binodal pressure of 0.3 GPa and $L_{\text{int}} = 3.3 \times 10^{16}$ W. The upper left panel shows pressure vs. depth in the molten interior, and positions for the ternary phase diagrams for three different depths in the planet. Ternary coordinates are in mole fractions. Reference bulk compositions projected into this ternary space are shown for Earth ($\oplus$), the model sub-Neptune interior ($\odot$), and Neptune ($\♆$).

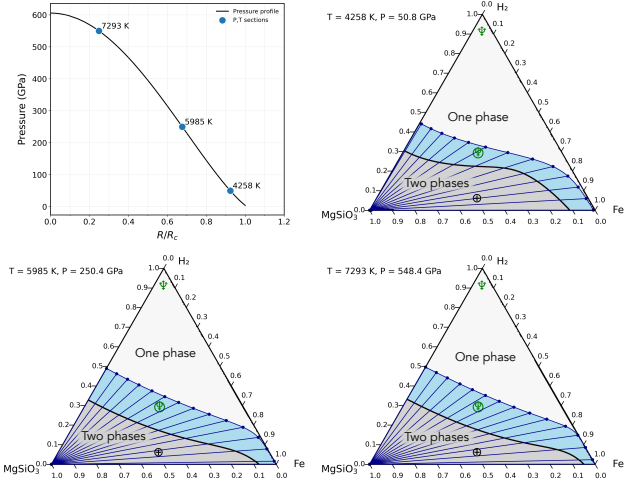


Figure 6. Phase diagrams for the interior of a model $6M_{\oplus}$ planet that has 3wt% total hydrogen with a binodal pressure of 4 GPa and $L_{\text{int}} = 1.6 \times 10^{15}$ W. The upper left panel shows pressure vs. depth in the molten interior, and positions for the ternary phase diagrams for three different depths in the planet. Ternary coordinates are in mole fractions. Reference bulk compositions projected into this ternary space are shown for Earth ($\oplus$), the model sub-Neptune interior ($\odot$), and Neptune ($\♆$).

world with a secondary atmosphere (R. Hu et al. 2024) and serves as a fiducial marker for such cases here. Similarly, planet LHS-1140 b lies in the transition between super-Earths and sub-Neptunes. It is tidally locked, with an orbital period of about 25 days and an equilibrium temperature of 230 K. It has been used as an example of a water world, with liquid water at the substellar point and perhaps a secondary atmosphere (C. Cadieux et al. 2024). Planets TOI-270 d and GJ 9827 d have lower bulk densities, placing them in the regime of volatile-rich sub-Neptunes (B. Benneke et al. 2024; C. Piaulet-Ghorayeb et al. 2024). Water has been detected in the atmospheres of both planets, albeit at different equilibrium temperatures. GJ 9827 d may host a high-mean-molecular-weight, steam-rich atmosphere (C. Piaulet-Ghorayeb et al. 2024),

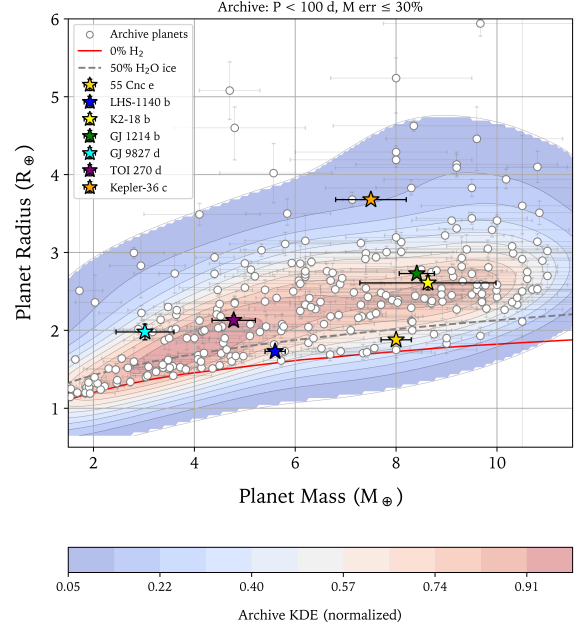


Figure 7. Mass vs. radius diagram for 261 archive planets, together with several example planets discussed in the text. The planets all have periods of <100 days and mass precision of less than or equal to 30%. The contours are for equal kernel density, illustrating the density of points. No points are shown below the pure Earthlike mass vs. density curve, since these planets are not included in our models. Also shown is a curve for 50% by mass water ice, based on an Earthlike core mixed with low-pressure water ice—see A. Agüichine et al. (2021).

while JWST observations of TOI-270 d reveal a hydrogen-dominated atmosphere with prominent absorption from CH_4 , CO_2 , and likely H_2O , as well as an inferred supersolar atmospheric metallicity (B. Benneke et al. 2024). Additional analyses report evidence for sulfur-bearing species in TOI-270 d’s atmosphere (L. Felix et al. 2025). The atmospheric properties of TOI-270 d have been interpreted either as signatures of interaction with an underlying magma ocean (M. C. Nixon et al. 2025) or as consistent with volatile-rich envelope scenarios, including liquid water configurations (M. Holmberg & N. Madhusudhan 2024). Interior structure analyses further explore whether such water-rich states are thermodynamically consistent with the observed bulk density and atmospheric composition (F. E. Rigby & N. Madhusudhan 2026). Planets GJ 1214 b and K2-18 b are squarely in the sub-Neptune field in mass versus radius space. GJ 1214 b is a “warm” sub-Neptune, with a period of just 1.6 days and an equilibrium temperature of ~ 500 – 600 K. It is thought to have a high-molecular-weight atmosphere, perhaps dominated by water, with a photochemical aerosol-rich cloud structure (P. Gao et al. 2023; E. M.-R. Kempton et al. 2023). K2-18 b, with an orbital period of 33 days around an M dwarf, exhibits a hydrogen-rich atmosphere with robust detections of CH_4 and CO_2 , and stringent upper limits on NH_3 abundance (N. Madhusudhan et al. 2023; R. Hu et al. 2025). Similar to TOI-270 d, these spectral constraints have been interpreted as consistent with either a relatively shallow H_2 -rich envelope potentially overlying a volatile-rich interior (the “Hycean” scenario; N. Madhusudhan et al. 2021) or with a thicker H_2 envelope in contact with a magma ocean (O. Shorttle et al. 2024). Last, planet Kepler-36 c serves as a fiducial sub-Neptune with an H_2 -/He-rich envelope, evidenced by its low

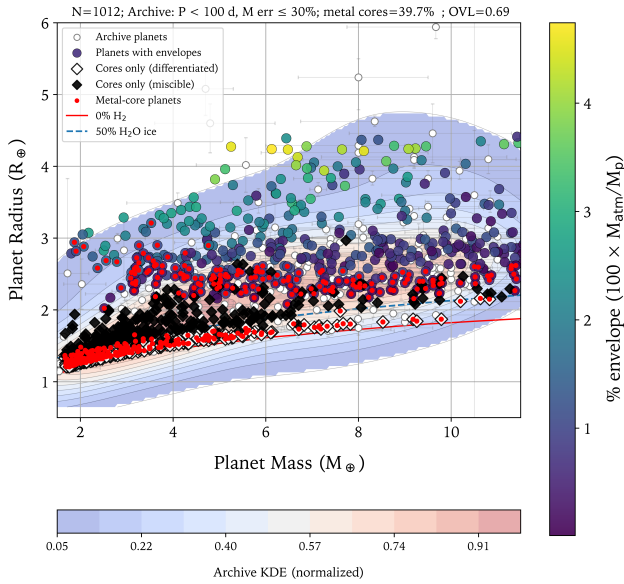


Figure 8. Comparison of model planets with observed planets in mass vs. radius space. Archive data, shown as the white circles with error bars, define contours of equal kernel density (KDE contours). The colors of the model data points reflect their mass fractions of envelopes (in percent), with the exception of the white diamonds. The colored circles with no red dots have entirely miscible interiors with H_2 -rich envelopes. The colored circles with red dots have H_2 -rich envelopes and have discrete Fe-rich metal cores. The black diamonds are fully miscible interiors that no longer have envelopes of hydrogen. The white diamonds with red dots are differentiated interiors with Fe-rich metal cores that have been stripped of their primary H_2 -rich envelopes. OVL is the overlap coefficient defined in the text.

density of just 0.87 g cm^{-3} (J. A. Carter et al. 2012). It orbits its star with a period of 16 days.

5.3. Random Draws

The 1012 random planets obtained in this study are plotted in mass versus radius space in Figure 8, together with the archive planets over the same mass range, shown also in Figure 7. The overall distributions are similar by eye, with all but three of the model planets falling inside the 5% normalized KDE contour for the archive data. The model planets concentrate in the same regions where the actual data are most prevalent (this is quantified below). Overall, 40% of the model planets have metal cores, and the vast majority of these have radii $< \sim 2.5 M_{\oplus}$.

The models produce four classes of planets. The least dense planets have relatively large mass fractions of hydrogen-rich envelopes and thus larger radii (the circles with colors representing the mass fractions of the envelopes in Figure 8). These planets have fully miscible interiors and thus no discrete, terrestrial-like metal cores. Related to these are hydrogen-rich planets that have hydrogen-rich envelopes and also have Fe-rich metal cores (the cores are denoted by the red dots in Figure 8). As one expects, these planets have somewhat smaller radii for a given mass. The third type of planet comprises those that are sufficiently hydrogen-rich that their interiors are fully miscible, but they have been stripped of their outer envelopes of hydrogen (the black diamonds in Figure 8). The fourth planet type consists of those that have discrete Fe-rich iron cores and no longer have discernible hydrogen-rich envelopes (the white diamonds in Figure 8). These are the closest to Earthlike in structure and

chemical makeup. Examples of the model planet structures are shown in Figure 9.

Broadly, the four groups cluster in ways analogous to the planet types described in Section 5.2. The model terrestrial-like super-Earths fall between the reference 0% rock/metal and 50% water curves in Figure 8. The region occupied by planets transitional between super-Earths and sub-Neptunes is populated by model planets with hydrogen-rich fully miscible interiors stripped of their envelopes, and by planets with H_2 -rich envelopes surrounding silicate and metal interiors with discrete Fe-rich metal cores. Model planets with fully miscible cores and significant H_2 -rich envelopes occupy the same mass–radius space as canonical sub-Neptunes with H_2 -/He-rich atmospheres.

The model and observed planet radii are compared in Figure 10. The latter have been corrected for completeness (see Appendix A) to bring the radius valley into relief and to facilitate comparison between the unbiased random samples and the actual data. The model data exhibit a probability density distribution very similar to the archive data corrected for observational biases, with peaks and a pronounced radius gap at the same radii. These features are mainly an inexorable result of atmosphere stripping.

The radius gap is known to decrease in radius with increasing period, and this feature is often used as an arbiter for successful models of planet formation and evolution (e.g., J. E. Owen & Y. Wu 2017; A. Gupta & H. E. Schlichting 2019). Model planets are compared with one estimate for the position of the radius gap as a function of orbital period by V. Van Eylen et al. (2018) in Figure 11. The model data exhibit a gap in radius that generally decreases with orbital period. This is a qualitative result, but is generally consistent with expectations from the actual data.

5.4. Distribution Comparison Metrics

In order to quantify the “chi-by-eye” apparent similarities between the synthetic planet population and the observed archive sample in mass–radius space, we use two statistics: a density overlap comparison using KDEs and an energy distance test calculated from the data points themselves.

The KDE overlap test provides a quantification of the visual overlap between the datasets (O. M. Eidous & E. A. Ananbeh 2024). The overlap coefficient, with a range from 0 to 1, is defined as

$$\text{OVL} = \int \min[f(x), g(x)] dx, \quad (64)$$

where $f(x)$ and $g(x)$ denote the KDEs of the model and archive distributions, respectively, and “min” in this context refers to the minimum of the two values (the overlap cannot exceed either distribution’s density at x). The two populations here are the archive data (A) and the model data (M).

The overlap coefficient measures the fraction of shared probability mass between two distributions, ranging from zero for no overlap to unity for identical distributions. It is sensitive to the overall spread and to various peaks and valleys in the distributions. Because finite sampling and observational scatter limit the maximum attainable overlap, we estimate an empirical reference value $\text{OVL}(A, A)$ by computing the overlap between random halves of the archive planet data themselves. We then define the

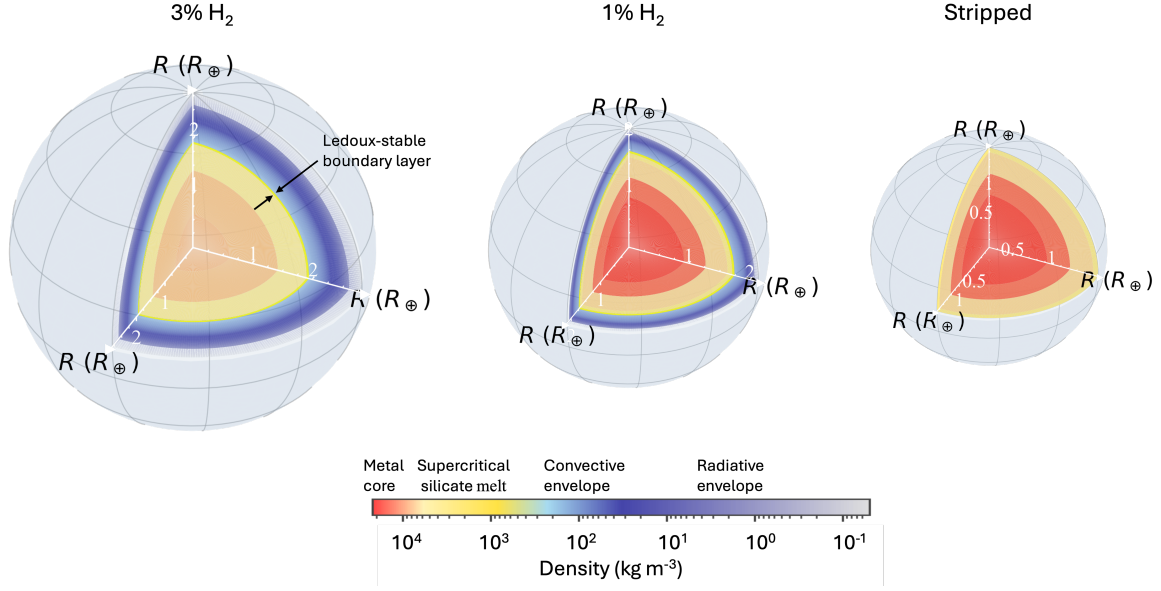


Figure 9. Three model planets illustrating the range of structures referred to in the text. Each planet has an initial mass of $6M_{\oplus}$ and a binodal pressure of 3.5 GPa. The bulk fraction of H_2 by mass is indicated. The planet on the right is the same as the 1% H_2 case but with the atmosphere stripped. The Ledoux-stable boundary layers are shown to scale, represented by the yellow bands at the bases of the H_2 -rich envelopes. The density contours indicate the metal core, the miscible silicate- H_2 -Fe interior, and the H_2 -rich envelopes.

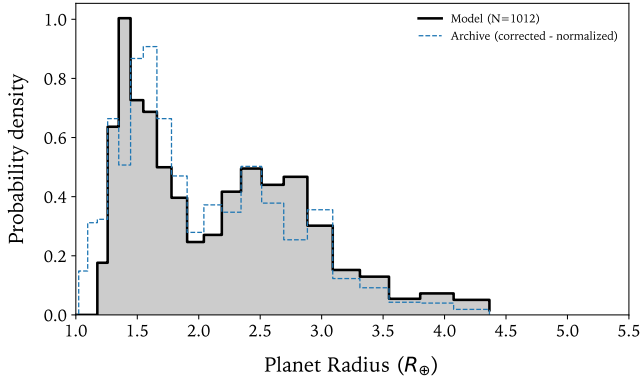


Figure 10. Histograms comparing model planet (gray filled) and observed planet (dashed line) radii. The latter are corrected for observational biases (see Appendix A).

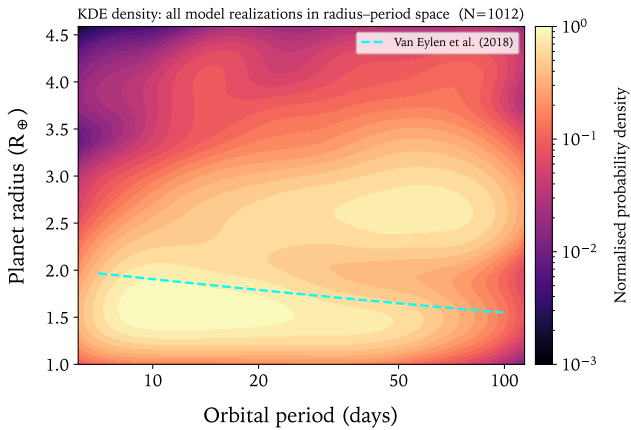


Figure 11. KDE plot of model planets in radius vs. period space showing a decrease in the model radius gap with orbital period. The position of the actual radius gap ca. 2018 as determined by V. Van Eylen et al. (2018) is shown for reference.

normalized overlap as the ratio

$$R_{\text{OVL}} = \frac{\text{OVL}(M, A)}{\text{OVL}(A, A)}, \quad (65)$$

which measures the fraction of observationally achievable overlap reproduced by the model population in view of the scatter among the data.

In this case, we obtain $\text{OVL}(M, A) = 0.69$ and $\text{OVL}(A, A) = 0.82$, implying $R_{\text{OVL}} \approx 0.84$. The model planets therefore reproduce 84% of the similarity achievable given the scatter in the actual planet data.

In order to further explore the statistical significance of the $\sim 20\%$ disparity between the archive data and the model data, we use their energy distance. The energy distance between two distributions is a scalar measure of their dissimilarity: equal to zero if and only if the distributions are identical, and increasing as they diverge. The name draws on the analogy with electrostatic potential energy: for example, just as the energy required to separate two charged bodies depends on the distance between them, so the energy distance between two distributions is constructed from the mean distances between points drawn from each in a “statistical potential energy.” The energy distance can be estimated directly from pairwise point distances without assuming any parametric form for the underlying distributions. We compute the energy distance between the model and archive samples $\{M_i\}$ and $\{A_j\}$ as follows (M. L. Rizzo & G. J. Székely 2016):

$$\mathcal{E}(M, A) = 2 \mathbb{E} \|M - A\| - \mathbb{E} \|M - M'\| - \mathbb{E} \|A - A'\|, \quad (66)$$

where $\mathbb{E}$ denotes the expected value, and $\|M - A\|$ is the Euclidean distance over all pairs; the expectations are averages over all pairwise Euclidean distances between sample points. The statistic vanishes entirely only when the two samples are drawn from the same underlying distribution. The statistical

significance is assessed using a permutation test in which the identities of data points are shuffled randomly for the pooled model–archive data. Each shuffled set is split into two sets and a value for $\mathcal{E}$ is evaluated, yielding the set $\mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_N\}$. The empirical p -value under the null hypothesis of a common underlying distribution for the model and archive data, representing the fraction of permuted statistics in $\mathcal{S}$ that exceed $\mathcal{E}_{\text{obs}}$, is then (B. Phipson & G. K. Smyth 2010)

$$p = \frac{1 + \sum_{k=1}^N \mathbf{1}[\mathcal{E}_k > \mathcal{E}_{\text{obs}}]}{1 + N}, \quad (67)$$

where $\mathbf{1}[\mathcal{E}_k > \mathcal{E}_{\text{obs}}]$ is 1 if the condition is satisfied and otherwise zero, and $\mathcal{E}_{\text{obs}} \equiv \mathcal{E}(M, A)$.

We obtain $\mathcal{E}(M, A) = 0.05$ with $p \simeq 2 \times 10^{-3}$ when comparing the model planets and the actual data (p is small but also resolution-limited, and thus it depends on N —a consequence of the small value for $\mathcal{E}(M, A)$). These values indicate that the two populations are statistically distinguishable (the null is rejected, given the small p), despite the similarity in their distributions visible in Figure 8.

Together, these tests indicate a substantial overlap between the model planets and the archive data, including the various peaks and valleys in the distributions, but because the samples are large enough to detect differences to high confidence, the small differences between the distributions are statistically resolved. We did not correct the archive data for observational biases in mass–radius space, because of the uncertainties involved, so some of the remaining differences are surely due in part to comparing data with no observational biases (models) to data with observational biases (archive).

6. Discussion

6.1. Planet Types

The randomly selected set of planets with structures governed by the degree of miscibility between silicates, iron metal, and hydrogen span the range of planet types exhibited by sub-Neptunes and super-Earths. To the extent that the model and archive planet populations are similar in their distributions in mass–radius and radius–period space, one can infer that the miscibility of hydrogen may be an important determining factor in planet formation in general over the relevant mass range of about 2 to $10 M_{\oplus}$.

The models are necessarily incomplete. For example, we have reduced the complex chemistry in these planets to a ternary system. In reality, the abundant hydrogen in their interiors will result in the formation of water or water-like species in the supercritical melts (M. Ikoma & H. Genda 2006; E. Kite & L. Schaefer 2021; H. E. Schlichting & E. D. Young 2022), and this water is expected to exchange with the dense H_2 -rich envelopes at equilibrium. The presence of water, while of great importance, is not expected to change the overall structure of our model planets. The ab initio molecular dynamics calculations upon which the hydrogen miscibility is based indeed demonstrate the conversion of H_2 to H_2O and OH-like species (T. Gilmore & L. Stixrude 2026) in the supercritical melt phase, consistent with macroscopic thermodynamic predictions at these conditions (H. E. Schlichting & E. D. Young 2022; A. Werlen et al. 2025a). Therefore, the existence of water in the interiors is “baked into” our calculations. Water that partitions from the supercritical melt

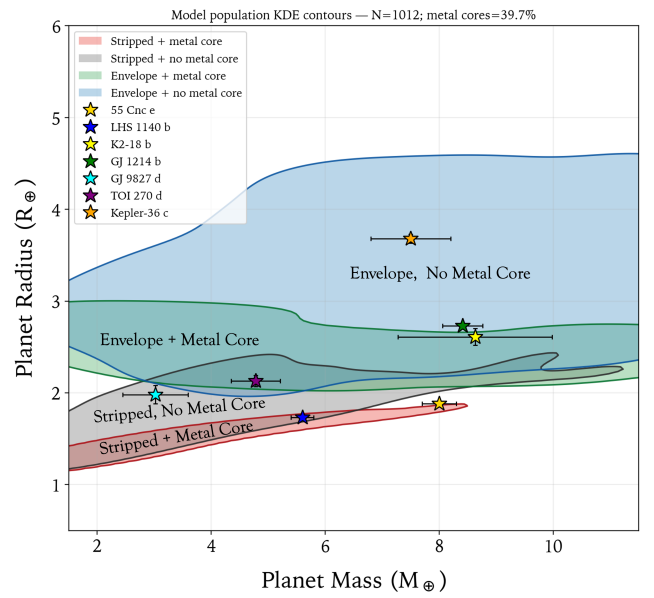


Figure 12. Comparison of fields for planet types defined by model planets with observed planets in mass vs. radius space. The regions associated with the four model groups are defined by KDE contours enclosing locations where the density exceeds 5% of the peak value for that group. The four groups are: (1) differentiated planets with metal cores stripped of their H_2 -rich atmospheres; (2) planets with no discrete metal cores and stripped of their H_2 -rich atmospheres; (3) differentiated planets with metal cores that retain their H_2 -rich envelopes, and (4) planets with no metal cores that retain their H_2 -rich envelopes.

to the hydrogen-rich envelope would slightly alter the mean molecular weight of the gas, but the change would be relatively minor (e.g., H_2O replacing half of the O_2).

In the planets where the envelopes are stripped, we have not included their subsequent evolution, implicitly assuming that their interiors become isolated with their full complement of hydrogen. We have therefore tacitly assumed that once the atmospheres are removed, rapid cooling quenches the system, preserving the H_2 inherited during the magma ocean phase. However, alternatively, cooling may be sufficiently slow that hydrogen degassing can occur before quenching (e.g., J. G. Rogers et al. 2025a). In addition, we do not try to model the geological evolution of planets stripped of their envelopes that leads to high-molecular-weight secondary atmospheres, including the outgassing of water produced in their interiors.

Despite these shortcomings, the model planet types appear to cluster in ways reminiscent of the different observed sub-Neptunes and super-Earths. It is therefore tempting to try to relate the variety of planets exemplified by the examples described in Section 5.2 and shown in Figure 7 to the models shown in Figure 8. To facilitate this comparison, the mass–radius fields defined by the four types of model planets together with the example observed planets are shown in Figure 12. The fields are defined by KDE contours for each group. Although these fields exhibit overlaps, they are nonetheless useful for guiding inferences about the nature of planets, based on their bulk densities in the context of hydrogen–silicate–iron miscibility.

Planets 55 Cancri e and LHS-1140 b are plotted in the “Stripped + Metal Core” field in Figure 12. This field corresponds to the white diamonds with red dots in Figure 8, representing differentiated bodies stripped of their primary atmospheres. These planets are basically pure rock and metal

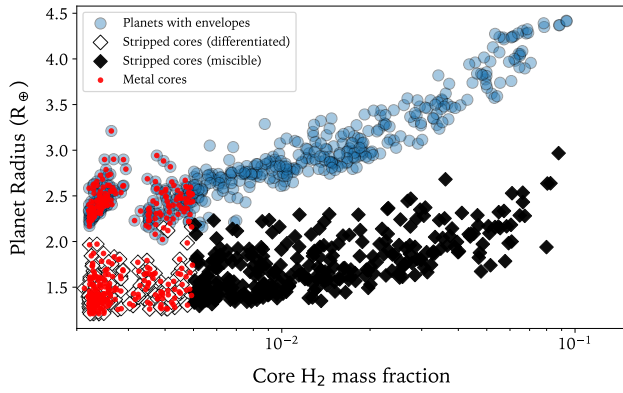


Figure 13. Plot of model planet radii vs. the mass fraction of H_2 in their interiors. The symbols are the same as those used in Figure 8. The sharp edge to the distribution at low mass fractions of $H_2 \sim 0.002$ is a reflection of the lower limit for numerically stable H_2 atmospheres in the models.

but with density deficits. The models here would suggest that the low densities of these planets compared with pure rock and metal, molten or solid, could be explained by significant mass fractions of hydrogen in their interiors (in whatever form). This is not unlike Earth, which is underdense relative to pure rock and iron metal, due to light elements, including H, in the metal core (H. E. Schlichting & E. D. Young 2022; E. D. Young et al. 2023).

Planets GJ 9827 d and TOI-270 d are plotted in the “Stripped, No Metal Core” and “Envelope + Metal Core” fields in Figure 12, representing the black diamonds and circles with red dots in Figure 8, respectively. Recall that GJ 9827 exhibits evidence for a steam-rich atmosphere, suggesting it may have lost its H_2 -rich envelope. This is consistent with its plot location in mass–radius space nearer to the “Stripped, No Metal Core” field. TOI-270 d, on the other hand, is thought to have retained an H_2 -rich envelope modified by interaction with a magma ocean. This is consistent with its plotting within the “Envelope + Metal Core” field in Figure 12. The models provide complementary, and not necessarily orthogonal, explanations for planets with broadly similar bulk densities: their cores may be undifferentiated and enriched in hydrogen, reducing the mean interior density without requiring massive hydrogen-rich envelopes. The model planets here do not preclude secondary, steam atmospheres in these cases, provided that such atmospheres do not significantly inflate the planetary radii.

Planets GJ 1214 b and K2-18 b fall squarely in the peak of the sub-Neptune probability density in mass–radius space, straddling the boundary between the fields that correspond to extant H_2 -rich envelopes with or without discrete metal cores in Figure 12. They therefore most resemble model planets with substantial H_2 -rich envelopes. However, recall that while K2-18 b is thought to have a hydrogen-rich envelope, GJ 1214 b may not. We note that the “Stripped, No Metal Core” field in Figure 12 extends toward these two planets, perhaps providing an explanation for GJ 1214 b not having a hydrogen-rich envelope.

Planet Kepler-36 c resembles the model planets with substantial H_2 -rich envelopes with mass fractions up to $\sim 4\%$ by mass and no metal cores (Figure 12). The models produce a significant range in H_2 atmosphere mass fractions at similar positions in mass–radius space (Figure 8).

The intrinsic degeneracies in mass–radius space that remain reflect the significant effects that large fractions of hydrogen in interiors can have on stripped cores, as shown in Figure 13. Our models underscore this effect in a systematic way.

6.2. Assumptions and Caveats

In our inverse epistemological approach, we have gone to some effort to reduce the assumptions about the formation of the model planets by, for example, mitigating the need to enforce initial intrinsic luminosities and justifying the prior distributions for input parameters with evolution models. However, a parameter not yet discussed in detail so far is the median bulk H_2 mass fraction prior to atmospheric escape used for our random draws. This parameter is, in effect, tuned to yield radii similar to the observed planets.

We show in Appendix B a simple calculation suggesting that accretion accompanied by the simultaneous fractional escape of hydrogen should lead to a log-normal distribution like that used in our models for reasonable estimates of parameters. In this simple calculation, the median value for planet mass fractions of H_2 of 1.8% corresponds to the accretion of protoplanetary bodies ranging in mass from 0.1 to $2.0M_\oplus$, with each accretion event resulting in the retention of between 18% and 100% of the primary atmospheres. The retention factor for each event is drawn at random from a parent population (Appendix B). The fact that the median mass fraction of H_2 for our model planets fits both the radius valley and the radius “cliff” (the dropoff in radii at about $4R_\oplus$) provides some a posteriori confidence in its significance. A more thorough modeling of the accretion process is warranted.

The relevant question raised by the initial conditions implicit in this study is whether accretion, including giant impacts in particular, results in the intermingling of hydrogen and molten rock. Several lines of evidence indicate that the mechanical and chemical co-mingling of hydrogen and silicate melt is likely. Three-dimensional smoothed-particle hydrodynamics (SPH) simulations show that individual giant impacts involving Earth-mass bodies rarely remove more than $\sim 10\%$ of H_2 –He primary envelopes comprising 5% of the targets, with cumulative losses over many impacts reaching 40% to 70% (M. J. Roche et al. 2025); large fractions of primary hydrogen accreted by protoplanets survive the violent accretion process. SPH typically suppresses material mixing across steep density contrasts and so cannot resolve the fine-scale entrainment of gas into melt. Nonetheless, M. J. Roche et al. (2025) infer that surviving volatiles are mostly dissolved in the silicate. Ab initio molecular dynamics simulations show that giant impacts drive portions of the colliding bodies to supercritical states, resulting in a continuous liquid–vapor structure (R. Caracas & S. T. Stewart 2023). In the resulting impact-generated “synestia,” volatiles are well mixed with the more refractory material (S. J. Lock et al. 2018). High-velocity impact simulations further show the vigorous mechanical mixing of primary envelope gas and vaporized rock followed by hydrogen–rock melt mixing (K. Kurosaki & S.-i. Inutsuka 2023; K. Kurosaki et al. 2023). More work is required on the fate of hydrogen during the accretion of protoplanetary materials, but thus far, substantial hydrogen in planet interiors seems to be a viable initial condition.

The connections to observations in this study are limited to comparisons in mass–radius and radius–period space and are therefore based on bulk density constraints alone. The physical

framework is restricted to hydrogen–silicate–iron phase equilibria and does not explicitly include additional volatile elements. Consequently, the models do not provide quantitative predictions for atmospheric compositions or spectral signatures. A direct comparison with transmission or emission spectra would require coupling the interior structure calculations to a more comprehensive chemical equilibrium framework, such as that of H. E. Schlichting & E. D. Young (2022), linked self-consistently to an atmospheric structure model (e.g., M. C. Nixon et al. 2025). Establishing this connection between interior phase equilibria and observable atmospheric chemistry remains an important next step.

7. Conclusions

Model planets based on the concept of variable miscibility between hydrogen, molten silicate, and molten iron metal, together with atmospheric escape, can reproduce roughly 80% of the occurrence density structure of sub-Neptunes and super-Earths in mass versus radius space. The model planet realizations also mimic the radius valley and radius versus period distributions of the actual planets. This overlap in the predicted occurrences with observations suggests that hydrogen–silicate–iron miscibility may serve as a unifying concept for planet formation for sub-Neptunes and super-Earths. In these models, the boundary between supercritical magma oceans and overlying hydrogen-rich envelopes is the binodal that specifies the equilibrium condition between the condensed phase and the atmosphere/envelope.

Planets modeled this way may explain the full range of planet types exhibited by sub-Neptunes and their potential super-Earth descendants. A key outcome is the distribution of hydrogen between planet interiors and their outer envelopes. Planets with hydrogen $< \sim 1\%$ by mass when they formed produce discrete, terrestrial-like metal cores, while planets born with more hydrogen are predicted to have interiors composed of a single phase, due to the effects of hydrogen. While necessarily incomplete, hydrogen–silicate–iron miscibility offers an overarching explanation for the wide variety of planet types among sub-Neptunes and super-Earths based solely on the accreted mass fractions of hydrogen-rich primary atmospheres and, importantly, the phase equilibria involving silicate, Fe metal, and H_2 .

Acknowledgments

We acknowledge financial support from NASA grant 80NSSC24K0544 (Emerging Worlds program). We are indebted to James G. Rogers (Institute of Astronomy, University of Cambridge), who contributed to the interior model Python code used in this study.

Appendix A

A.1. Data Extracted from the NASA Archive

The exoplanet sample used in this work was obtained from the NASA Exoplanet Archive with the Table Access Protocol (TAP) synchronous interface, querying the `pscomppars` table. Our query retrieved the planet mass, radius, orbital period, mass provenance flags, and host-star properties including stellar mass and stellar age with associated uncertainties. We selected planets with reported masses within

a specified interval from 1.5 to 11.5 $M_\oplus$, in keeping with our model mass range. We set a maximum orbital period of 100 days. Planets were further required to be associated with usable uncertainty estimates, meaning both M_p and R_p must have reported one-sided uncertainties. A quality cut was applied based on the fractional mass precision:

$$\frac{\sigma_M}{M_p} \leq \epsilon_M = 0.3. \quad (A1)$$

In order to account for asymmetric errors, the adopted fractional uncertainty is taken to be

$$\sigma_M = \max(|\sigma_{M,+}|, |\sigma_{M,-}|), \quad (A2)$$

so that the most conservative reported uncertainty was used.

In the present work, we include all observationally determined masses reported in the archive, including measurements derived from radial velocity (RV) analyses and transit-timing variations (TTVs), while excluding masses inferred from empirical mass–radius relations; only dynamically or observationally measured planet masses are retained.

A.2. Pseudo-completeness Correction for the Radius Distribution

A completeness-corrected planet radius distribution was created to enable comparison between observed exoplanet samples and model populations. The method provides an approximate occurrence-like quantity (planets per star per radius bin) without performing a full injection–recovery analysis.

Planet properties (R_p , P) and host identifiers are obtained from the NASA Exoplanet Archive along with their corresponding host-star properties, including radius R_* , mass M_* , observing baseline T_{obs} , duty cycle d (the fraction of the observing baseline during which photometric data were collected), and the Combined Differential Photometric Precision (J. L. Christiansen et al. 2012)—a per-star photometric noise metric evaluated at a reference transit duration. These quantities define the detectability for hypothetical planets around each target star.

Planets are grouped into radius bins $\mathcal{B}_j$. Each bin is assigned a representative radius R_j and a representative orbital period P_j , taken as the median period of the planets within the bin. This preserves the dependence of the detection efficiency on orbital period while maintaining a one-dimensional radius distribution.

For a planet of radius R and period P orbiting a star, the transit depth is

$$\delta = \left(\frac{R}{R_*} \right)^2. \quad (A3)$$

Assuming circular central transits, the duration is approximated by

$$T_{14} \approx \frac{P R_*}{\pi a}, \quad a = \left(\frac{GM_* P^2}{4\pi^2} \right)^{1/3}, \quad (A4)$$

where T_{14} denotes the total transit duration, defined as the interval between the first and fourth contact—i.e., when the planetary and stellar limbs first and last become tangent.

The expected number of observed transits is

$$N_{\text{tr}} = d \frac{T_{\text{obs}}}{P}. \quad (\text{A5})$$

From these values, we obtain the photometric noise scaled with transit duration as $\sigma(T_{14}) \propto T_{14}^{-1/2}$, yielding an approximate signal-to-noise ratio accumulated over multiple transit events—i.e., the multiple-event statistic (MES) (J. M. Jenkins 2002):

$$\text{MES}(R, P|s) \simeq \frac{\delta\sqrt{N_{\text{tr}}}}{\sigma(T_{14})}. \quad (\text{A6})$$

The detection efficiency can be modeled using a sigmoidal threshold function based on the MES (C. J. Burke et al. 2015):

$$\epsilon(\text{MES}) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\text{MES} - \mu}{\sqrt{2}\sigma} \right) \right], \quad (\text{A7})$$

with $(\mu, \sigma) = (8, 1.5)$ —values calibrated to the Kepler pipeline detection threshold (J. M. Jenkins et al. 2010; C. J. Burke et al. 2015).

For each radius bin $\mathcal{B}_j$, the effective number of searchable stars is

$$D_j = \sum_s \mathbf{1}[N_{\text{tr}}(P_j|s) \geq N_{\text{tr,min}}] \epsilon(\text{MES}(R_j, P_j|s)), \quad (\text{A8})$$

where $N_{\text{tr,min}} = 3$ is the minimum number of observed transits required for a detection threshold consistent with the Kepler pipeline (J. M. Jenkins et al. 2010).

If $\mathcal{P}_j$ denotes planets with radii within bin $\mathcal{B}_j$, the occurrence per star per bin is

$$f_j = \frac{|\mathcal{P}_j|}{D_j}. \quad (\text{A9})$$

We note that this expression does not include a correction for the geometric transit probability $p_{\text{tr}} = R_*/a$; the quantity f_j should therefore be interpreted as a detection-efficiency-corrected count rather than a true occurrence rate (see A. W. Howard et al. 2012; F. Fressin et al. 2013). In practice, each planet is assigned a weight

$$w_{\text{occ}} = \frac{1}{D_j}, \quad (\text{A10})$$

and the radius distribution is obtained by summing these weights within bins. Summing the weights of the planets within a bin reproduces the occurrence estimate:

$$\sum_{i \in \mathcal{P}_j} w_{\text{occ}} = f_j. \quad (\text{A11})$$

Appendix B

B.1. Hydrogen Accretion

The observed spread in the inferred H_2 mass fractions among sub-Neptunes is well described by the log-normal distribution used in our models with a median value of 1.8% by mass. In lieu of a detailed and highly uncertain accretion model, here we describe a simple stochastic numerical accretion experiment that provides a physical motivation for such a distribution. The goal is not to construct a full accretion

model, nor to fit the adopted distribution perfectly, but rather it is to demonstrate that log-normal distributions of H_2 fractions arise naturally from the combination of discrete accretion events and partial losses of envelopes surrounding the accreting bodies in general.

In the experiment, planets are assembled by the sequential addition of smaller solid bodies drawn at random from a prescribed mass distribution until a target core mass is reached. Each accreted body has mass m_i , and the total solid mass is the sum of these contributions. During a sequence of accretion events, two processes occur recursively. First, the impact associated with each accretion event partially strips the preexisting H_2 envelope; after event i , only a fraction x_{r_i} of the envelope mass present prior to the next collision survives. Second, the newly accreted body contributes additional H_2 according to the cooling-limited scaling of S. Ginzburg et al. (2016), in which the envelope mass fraction associated with a body of mass m scales approximately as

$$f = 0.02 \left(\frac{m}{M_{\oplus}} \right)^{0.8} \left(\frac{T}{1000 \text{ K}} \right)^{-1/4} \left(\frac{t_{\text{disk}}}{1 \text{ Myr}} \right)^{1/2}, \quad (\text{B1})$$

where $M_{\oplus}$ is one Earth mass, T is the characteristic disk temperature at the planet’s location, and t_{disk} is the gas disk lifetime. The increment in the growing H_2 -rich envelope mass during each accretion event is then $\Delta M = f(m)m$. The envelope mass therefore evolves according to

$$M_{\text{env}}^{(k+1)} = x_{r_k} M_{\text{env}}^{(k)} + \Delta M_k, \quad (\text{B2})$$

where ΔM_k is the increment of H_2 accreted during event k . The final H_2 mass fraction is then

$$f_{\text{H}_2} = \frac{M_{\text{env}}}{M_{\text{core}} + M_{\text{env}}}. \quad (\text{B3})$$

The essential statistical feature of the model is the multiplicative survival factor x_{r_k} . Physically, x_{r_k} represents the combined effects of impact-driven stripping, shock heating, transient hydrodynamic escape, or variations in cooling efficiency. We envision the total retention factor during a given accretion event schematically as a product of independent contributions. Provided that these individual contributions have finite variance and are not dominated by a single catastrophic process, the central limit theorem implies that the logarithm of the net retention factor is approximately Gaussian in its distribution.

Iterating Equation (B2) through N accretion events gives

$$M_{\text{env}}^{(N)} = \left(\prod_{j=0}^{N-1} x_{r_j} \right) M_{\text{env}}^{(0)} + \sum_{k=0}^{N-1} \left[\Delta M_k \prod_{j=k+1}^{N-1} x_{r_j} \right], \quad (\text{B4})$$

so that the final envelope mass is the sum of the initial envelope mass and all subsequent accretion increments, each weighted by the product of the retention fractions associated with later impacts.

While M_{env} is not a pure product, its value is strongly influenced by accumulated multiplicative retention factors. When many stochastic accretion events contribute comparably and no single increment dominates the total budget, the products $\prod x_{r_j}$ introduce effectively random variability into the hydrogen accretion process. Under these conditions, $\ln M_{\text{env}}$ behaves approximately as a sum of random contributions associated with $\ln x_{r_j}$ and modestly varying accretion terms.

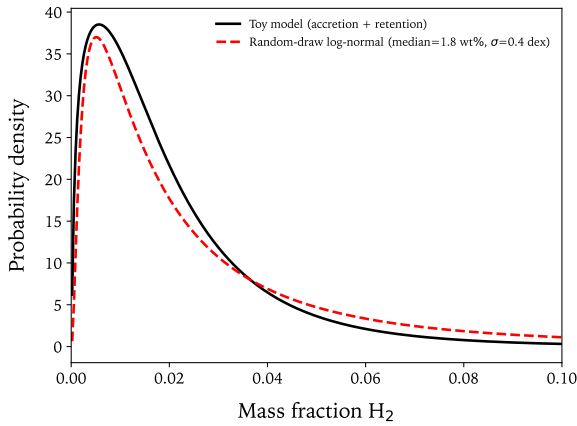


Figure 14. Distribution of hydrogen mass fractions produced by the simple accretion calculation for 30,000 planets each with a total mass of $5M_{\oplus}$. The distribution used to generate the model planets in this study is shown for comparison (dashed curve).

The distribution of M_{env} consequently approaches a log-normal distribution. Since the final core mass is fixed by construction in this experiment, the H_2 mass fraction inherits this approximately log-normal distribution. The log-normality is not imposed a priori. Rather, it emerges from the combined effects of stochastic growth and partial loss of hydrogen during accretion events.

Figure 14 shows the results from this calculation for a range of accreting masses from 0.1 to $2.0M_{\oplus}$ and an envelope retention fraction x_{r} drawn from a log-normal distribution defined by $\ln r \sim \mathcal{N}(\mu_{\log} = 0.8, \sigma_{\log} = 1.5)$, corresponding to a range of retention fractions from 0.18 to the maximum value of unity. The curve resembles the distribution used to generate the model planets (Figure 14).

This simple calculation illustrates that a log-normal distribution of H_2 mass fractions can arise naturally as a statistical consequence of hierarchical accretion combined with incremental losses of hydrogen for a reasonable set of parameters. In this sense, the adopted log-normal description of sub-Neptune H_2 mass fractions in our models can be viewed as an emergent property of stochastic assembly.

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